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Thermodynamics and Fluid Mechanics 2

Fluids Topic 5: Turbomachinery

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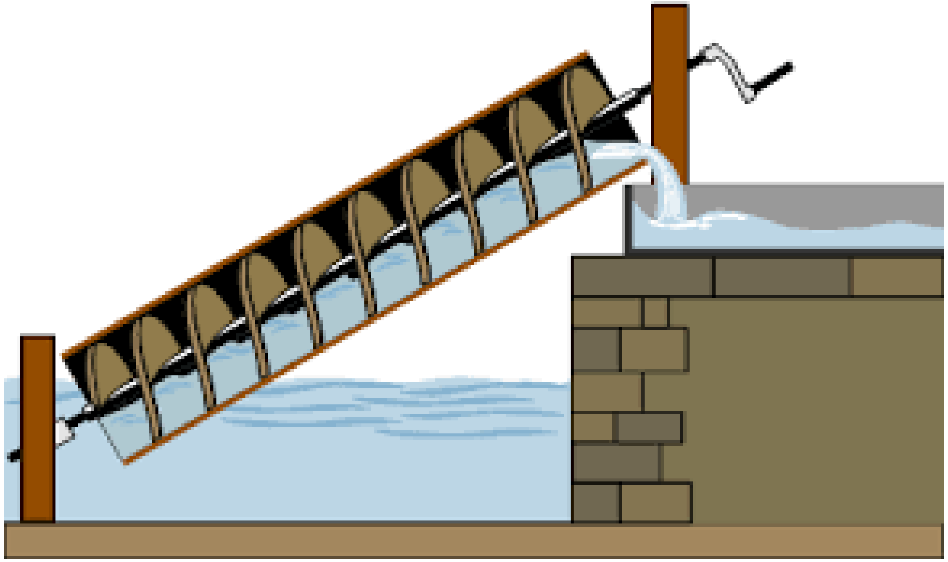
Topic 5 – Turbomachinery

From Latin, *turbo*- means ‘spin’ or ‘whirl’.

Turbomachinery: rotating devices that add (pump, fan, compressor) or extract (turbine) energy from a fluid.

Norias of Hama: (Hama, Syria; 1000 BC) rotating wheels that scoop water from a river to an aqueduct
https://en.wikipedia.org/wiki/Norias_of_Hama



Archimedes screw pump (300 BC)
http://mechstuff.com/amazing-archimedean-screw/

Topic 5 – Turbomachinery

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Turbomachinery: rotating devices that add (pump, fan, compressor) or extract (turbine) energy from a fluid.

Dutch windmill: used to pump water or crush corn, cereals, etc.

https://thekidshouldseethis.com/post/770815990372
(it’s a video)

Wind turbines, extract energy from wind to generate electricity

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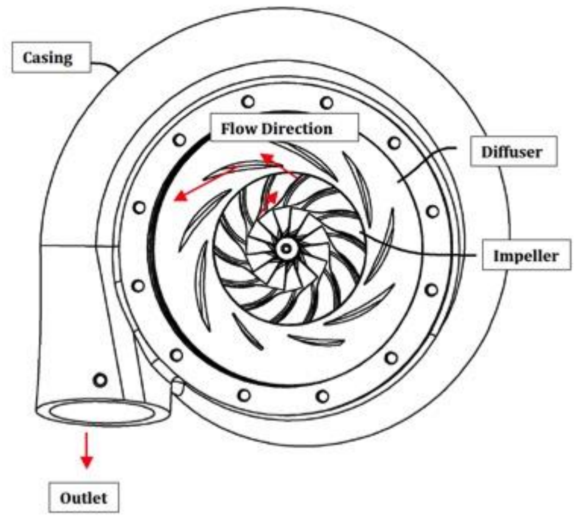
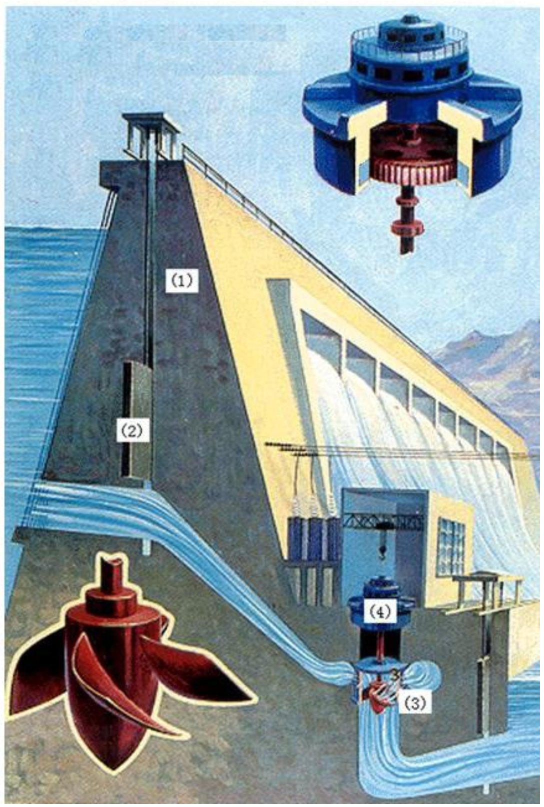
Centrifugal compressor



Computer cooling fan



Propeller turbine

Propeller of a cruise ship

Topic 5 – Turbomachinery

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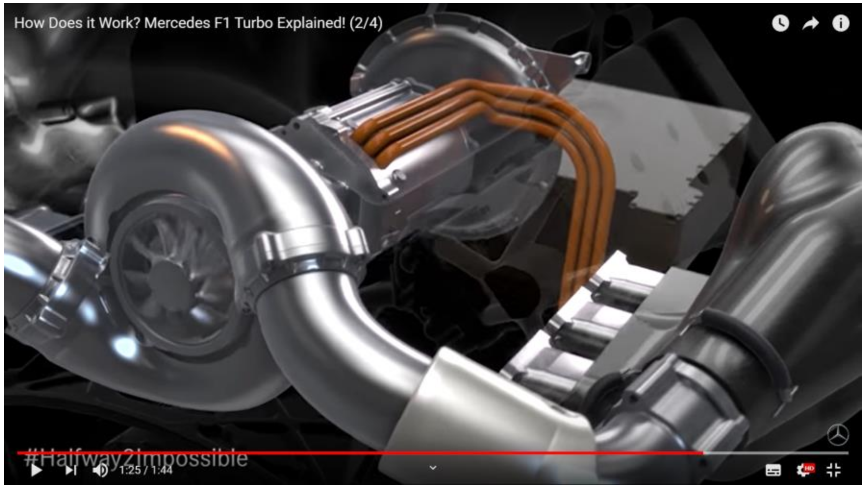
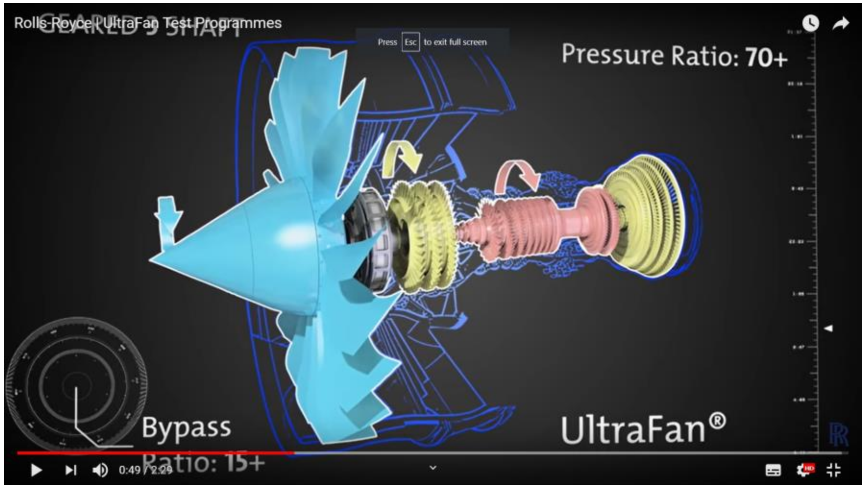


Mercedes F1 turbocharger: expands exhaust gases in a turbine to drive a compressor and pressurize air at engine intake

https://www.youtube.com/watch?v=l2_eTLCz_3I



Rolls-Royce UltraFan engine: expands exhaust gases in a turbine to drive a compressor and pressurize air at engine intake and generate high speed gases for propulsion

https://www.youtube.com/watch?v=auk-LRqtFs8

Topic 5 – Turbomachinery

- Introduction and classification
- Positive displacement pumps
- Dynamic pumps
- Integral analysis of centrifugal pump
- Centrifugal pump performance
- Cavitation and NPSH
- Dimensional analysis and similarity for pumps
- Mixed and axial flow pumps
- Reaction and impulse turbines
- Dimensional analysis and similarity for turbines
- Wind turbines

Topic 5 can be studied in F. White, Ch. 11

Topic 5 – Turbomachinery

Learning outcomes:

- Know the difference between devices that *add* energy to a fluid and those that *remove* energy
- Know the two difference between the two main *pump classifications* (positive displacement and dynamic)
- *Positive displacement pumps.*
  - Be able to name, sketch and describe the operation of the main types of PD pumps
  - Be able to give the main advantages and disadvantages of PD pumps
- *Dynamic pumps.*
  - Be able to name, identify and explain the function of the main parts of a centrifugal pump (eg impeller, eye, outlet pipe etc)
  - Be able to sketch and name the three blade types for a centrifugal pump and know their advantages/disadvantages
  - Recognise and be able to use the expression for power delivered to the pump fluid (water power, $P_w = \rho Q g H$)
  - Recognise and be able to use the expression for pump input power ($P = \omega T$)
  - Recognise and be able to use the expression for pump efficiency

Topic 5 – Turbomachinery

Learning outcomes:

- *Dynamic pumps (continued).*
  - Be able to write down and explain the main causes of inefficiency in a centrifugal pump
  - Be able to sketch and describe all the main features of a pump performance curve
  - Be able to read data from a manufacturer's pump performance chart (flowrate, head, efficiency, NPSH)
  - Be able to describe the cause and main effects of cavitation in a pump
  - Be able to perform NPSH calculations using EBE
  - Recognise and be able to use the 5 pump non-dimensional groups
  - Recognise and be able to use pump similarity rules

- *Mixed and axial flow pumps*
  - Know when a mixed or axial flow pump might be the best choice
  - Be able to calculate specific speed
  - Be able to use the specific speed chart to work out which pump type is best for a given application

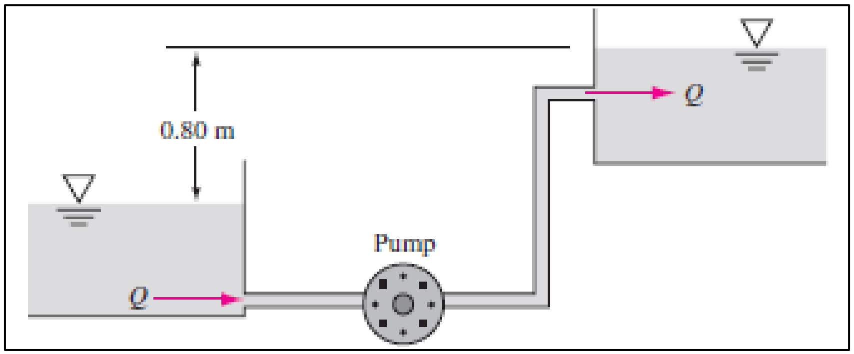
Learning outcomes:

- *Turbines*
  - Be able to articulate the main difference between reaction and impulse turbines
  - For reaction turbines be able to explain where radial, mixed and axial flow configurations are best
  - Recognise and be able to use the turbine non-dimensional groups
  - Be able to calculate power specific speed
  - Be able to use the power specific speed chart to determine the most appropriate turbine type

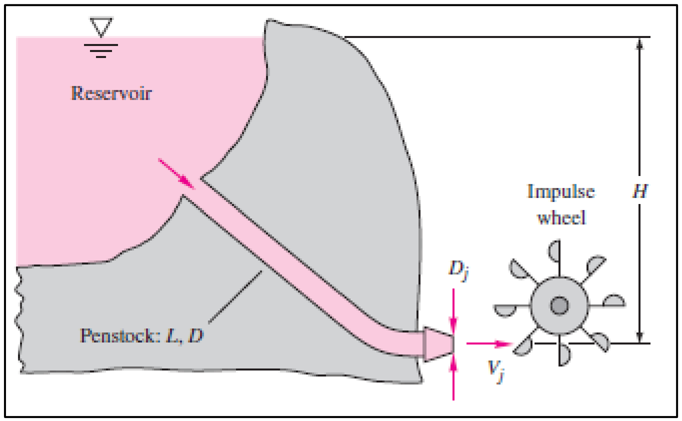
- *Wind turbines*
  - Be able to sketch and describe HAWT and VAWT
  - Be able to articulate the main advantages and disadvantages of HAWT and VAWT
  - Be able to explain using the Betz actuator disk approach with equations, diagrams and text why there is a maximum power coefficient for a wind turbine
  - Know $C_{P,max}$ for an ideal, frictionless wind turbine

Introduction and classification

Pump: a machine that adds energy to the fluid, e.g. to increase its potential energy



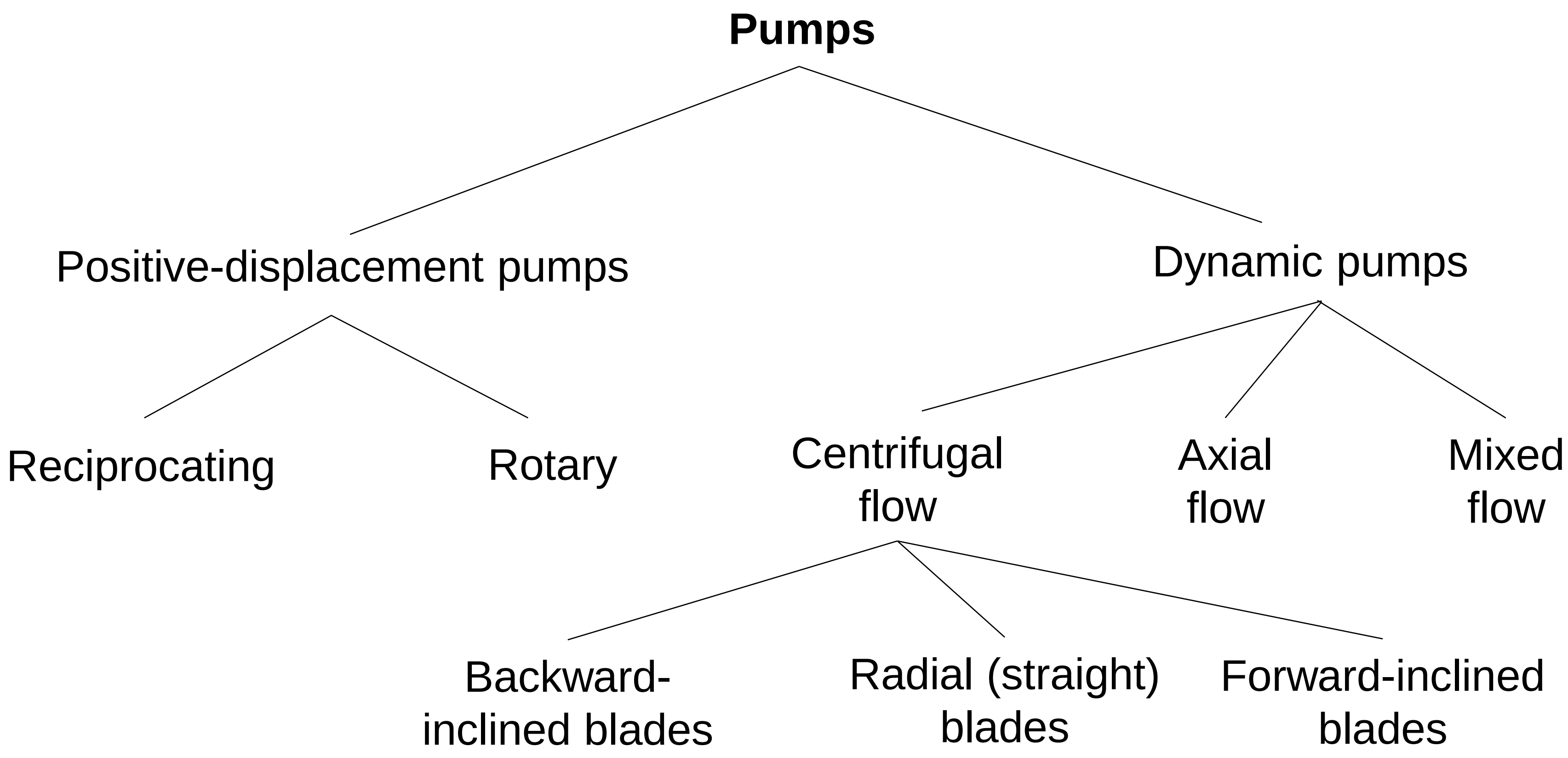
Turbine: a machine that extracts energy from the fluid, e.g. to drive an electrical generator



- 'Pump' is the name used when the operating fluid is a liquid
- If the fluid is a gas, different terms are used depending on the pressure rise achieved: *fan*, for very small pressure rise ($< 10^{-2}\,bar$); *blower*, for intermediate values ($< 1\,bar$); *compressor*, for pressure rise $> 1\,bar$.

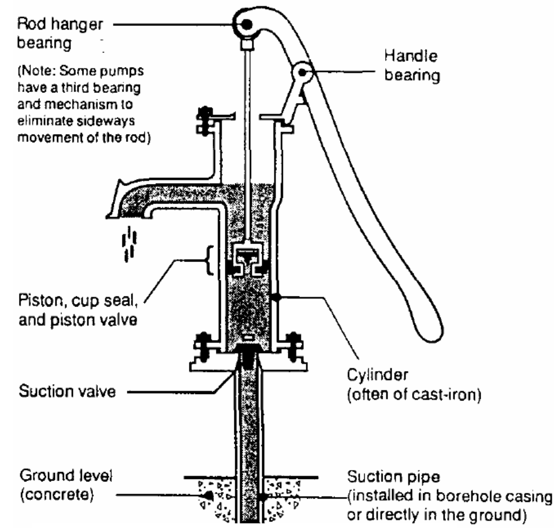
In TF2, we will consider pumps and turbines working with liquid, where density changes are small and therefore incompressible flow theory can be used.

Introduction and classification

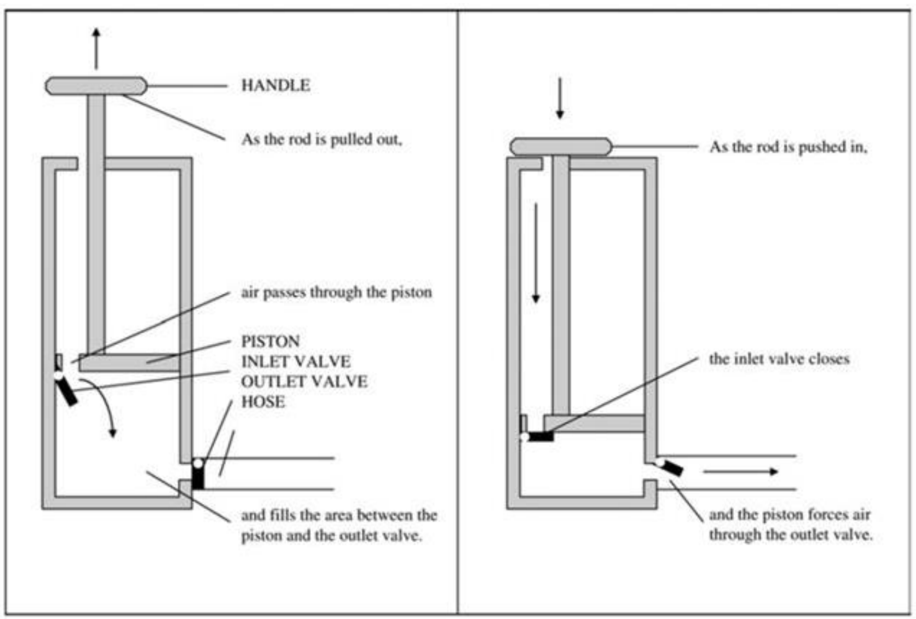


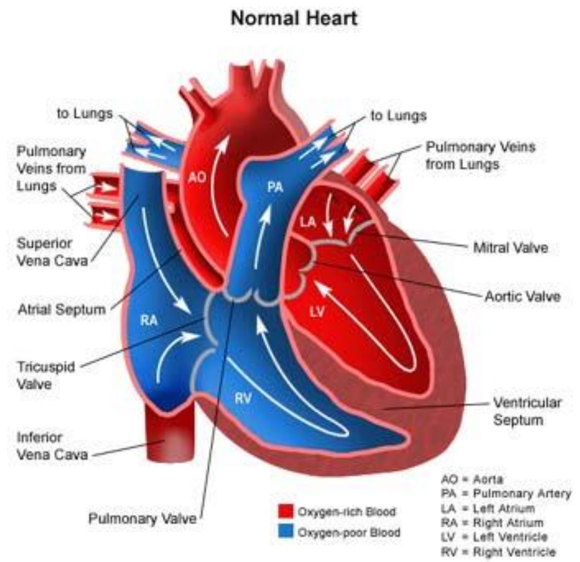
Positive displacement (PD) pumps

- PD pumps force the fluid along by volume changes: a cavity opens, drawing liquid into a chamber; the cavity then closes and the fluid is squeezed through an outlet.
- Examples are hand water pumps, bicycle pumps, or the mammalian heart



Hand water pump: https://www.youtube.com/watch?v=UmZYQ5I0CDU

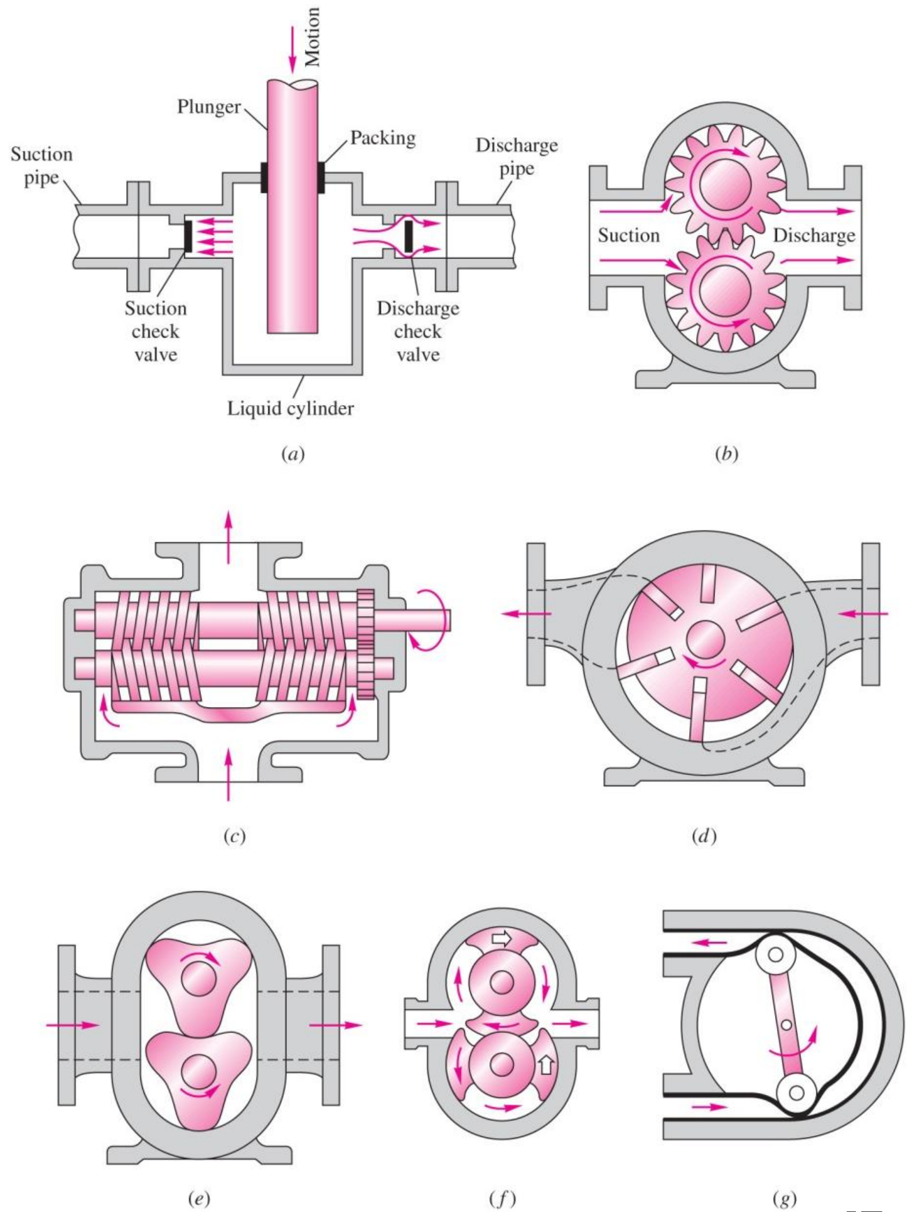




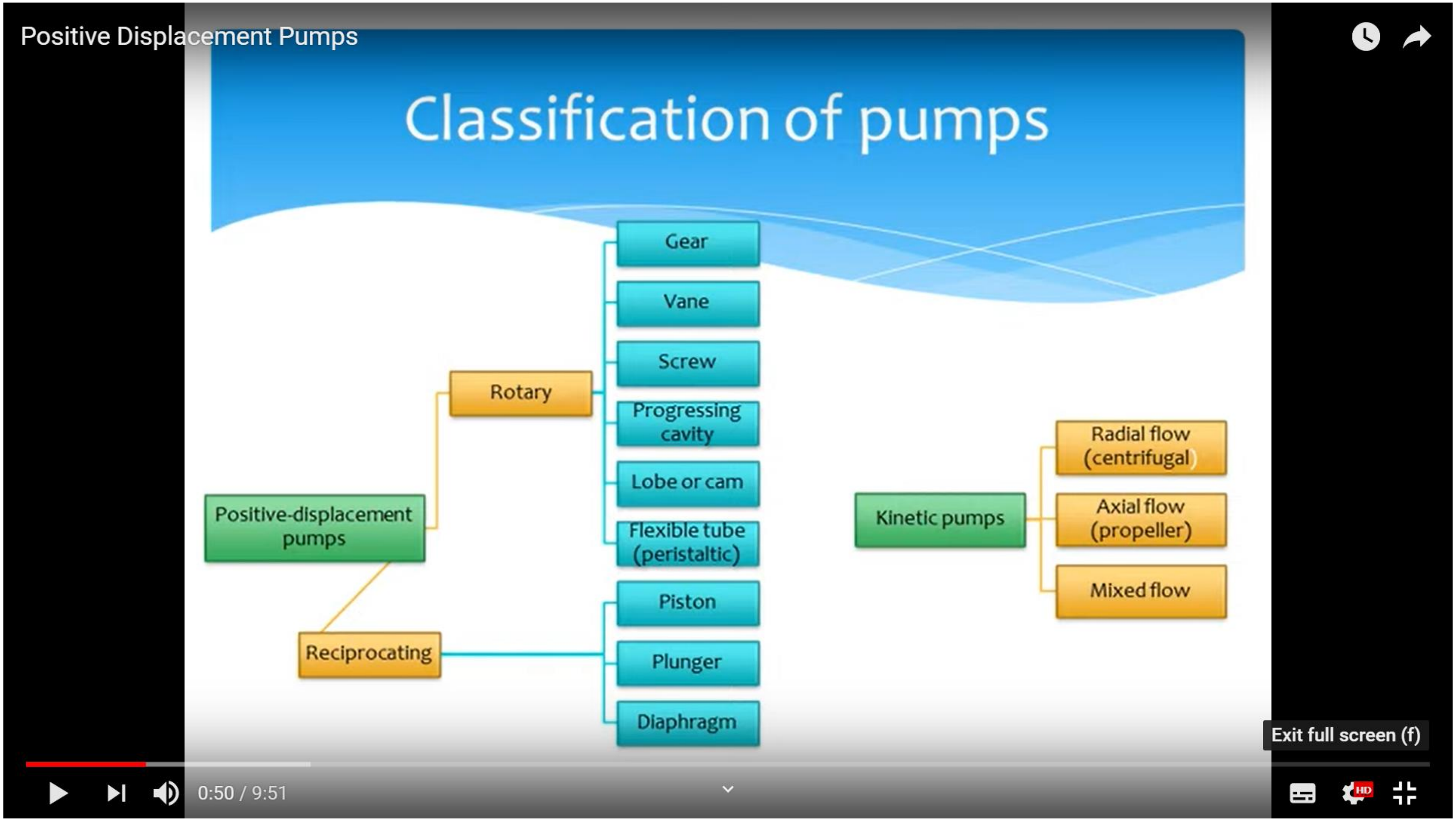
Positive displacement (PD) pumps

Classification of PD pumps (from F. White):

- Reciprocating
  - Piston or plunger (a)
  - Diaphragm
- Rotary
  - Gear (b)
  - Screw (c)
  - Sliding vane (d)
  - Lobe (e)
  - Double-circumferential piston (f)
  - Flexible-tube squeegee (g)

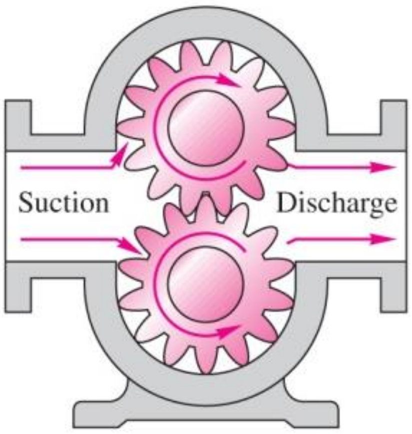


Positive displacement (PD) pumps

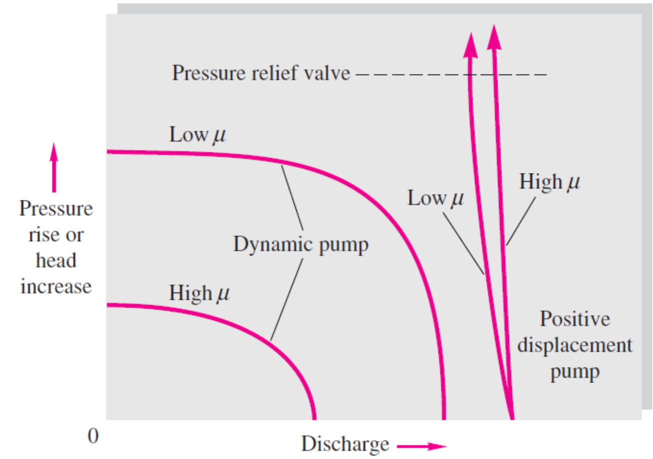


https://www.youtube.com/watch?time_continue=1&v=U3KWi6vAYFM&feature=emb_logo

Positive displacement (PD) pumps



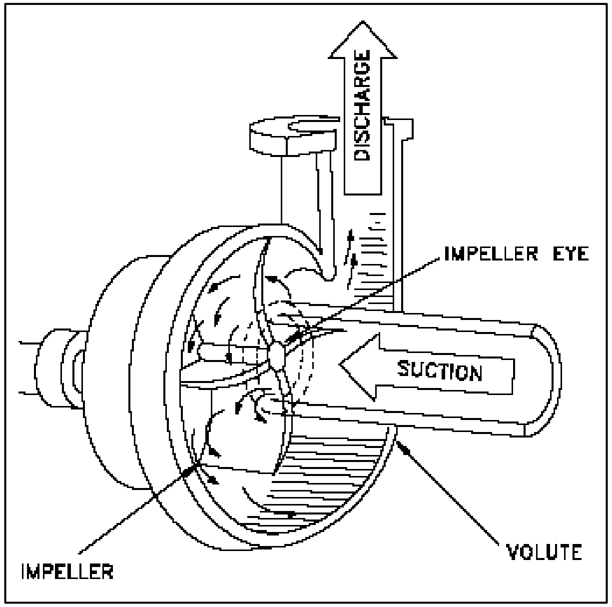
- PD pumps force the fluid along by volume changes: a cavity opens, drawing liquid into a chamber; the cavity then closes and the fluid is squeezed through an outlet.
- All PD pumps deliver a pulsating/periodic flow.
- They deliver any fluid regardless of its viscosity (dynamic pumps struggle with very viscous fluids).
- They are self-priming: the operating fluid will automatically fill the chamber (whereas dynamic pumps need priming).
- They can operate up to very high pressures (300 atm), but low flow rates (25 m^3/h), as opposed to to dynamic pumps.
- The flow rate (discharge) of a PD pump can only be changed by varying the displacement or the speed.



Dynamic pumps

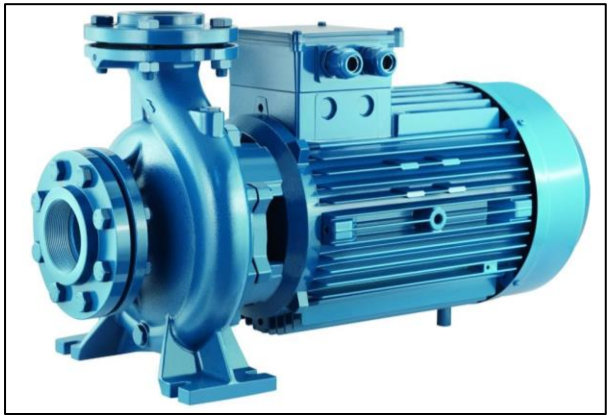
Dynamic pumps add momentum to the fluid by means of fast-moving blades or vanes.

- They are classified based on the direction of the flow at the exit: centrifugal (radial), axial, mixed flow. We will mostly focus on centrifugal pumps.
- In contrast with PD pumps, there is no closed volume: the fluid increases momentum while moving through open passages and then converts its high velocity to a pressure increase by exiting into a diffuser section.
- They provide very high flow rates (up to 70000 m^3/h) but usually with moderate pressure rises (a few atm).
- They need *priming*: if filled with gas, they cannot suck up the liquid into their inlet.



Schematic of a centrifugal pump

A commercial centrifugal pump



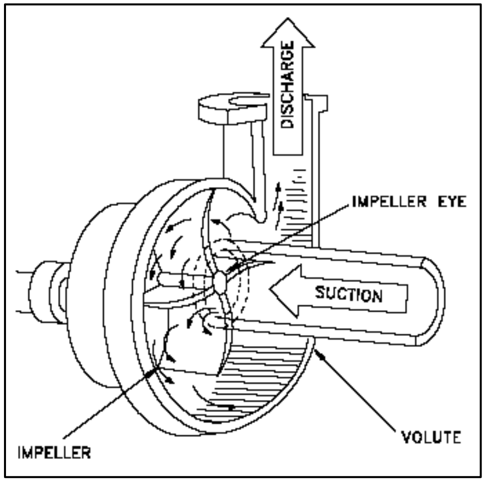
Centrifugal pumps

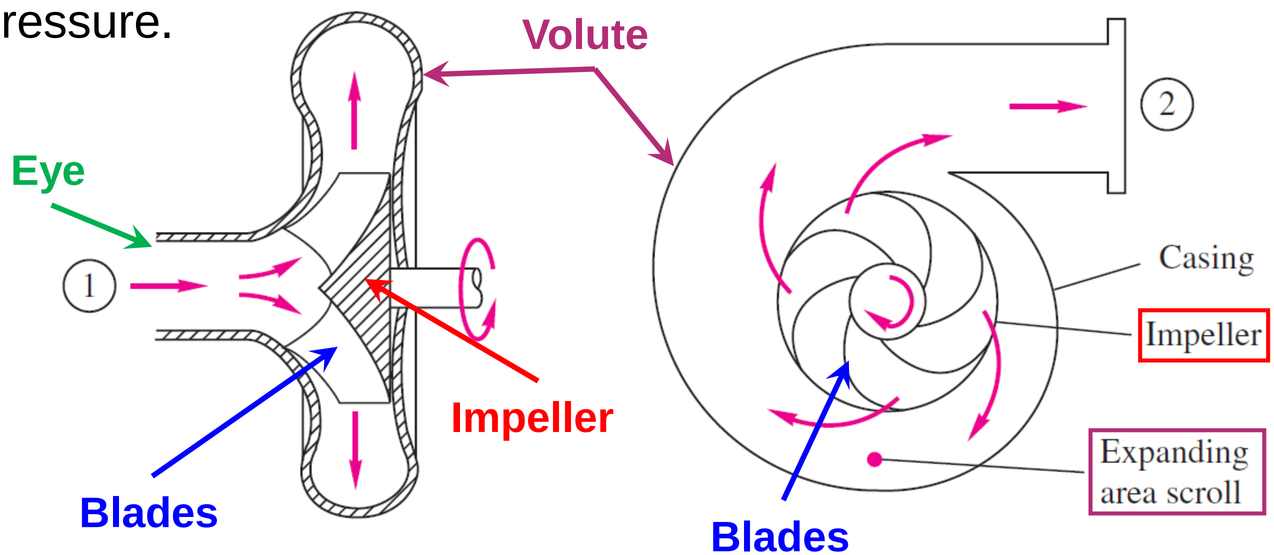
The centrigual pump consists of an impeller (the rotor) rotating within a casing (the stator).

The fluid enters axially through the eye of the pump casing, is caught up by the impeller blades and whirled tangentially and radially outward, until it leaves via the expanding area section of the casing, also called diffuser or volute.

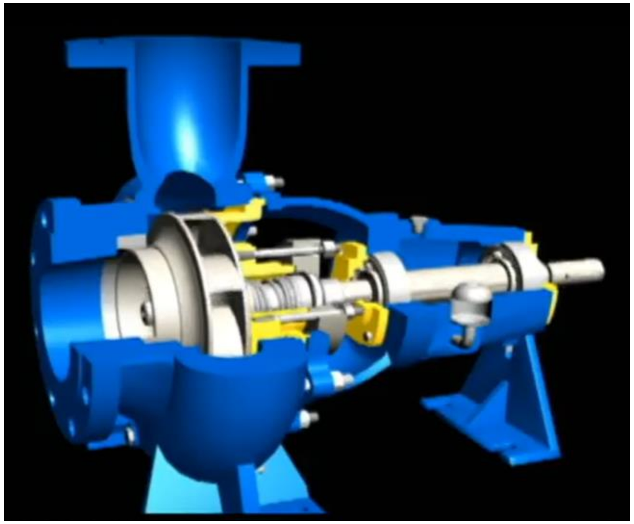
The flow passages between the blades and in the volute are diverging: this decelerates the flow and further increases its pressure.

Schematic of a centrifugal pump





Centrifugal pumps



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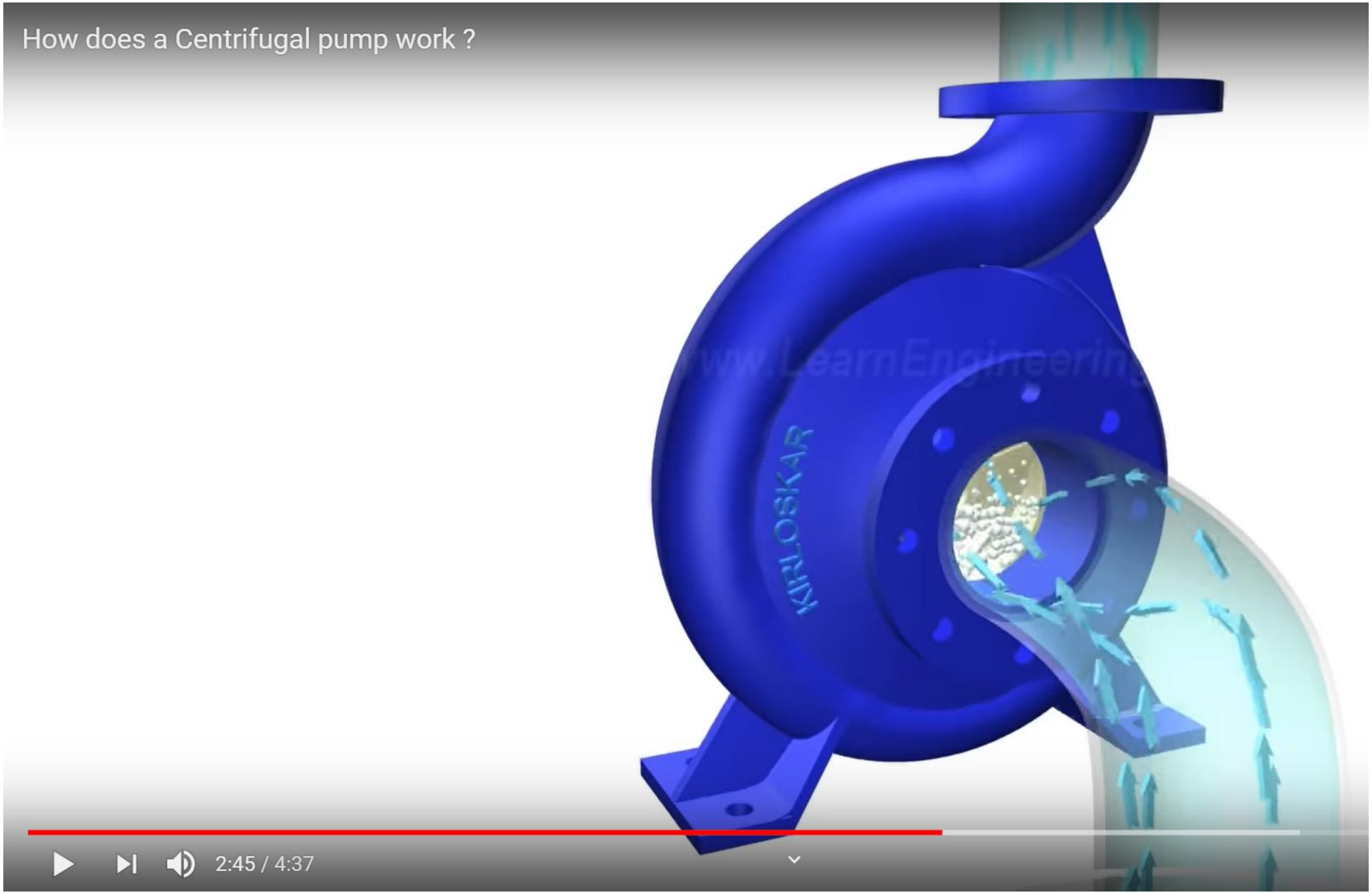
Centrifugal pumps



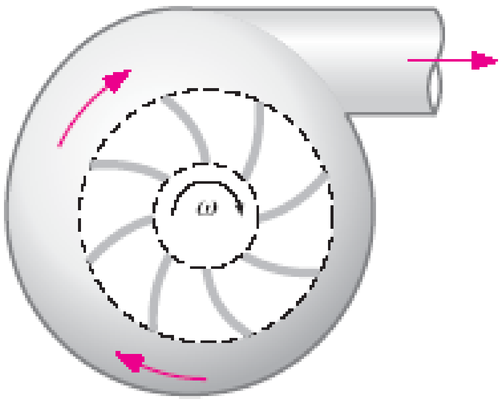
https://www.youtube.com/watch?time_continue=1&v=BaEHVpKc-1Q&feature=emb_logo

Centrifugal pumps blades

Classification based on blades orientation.

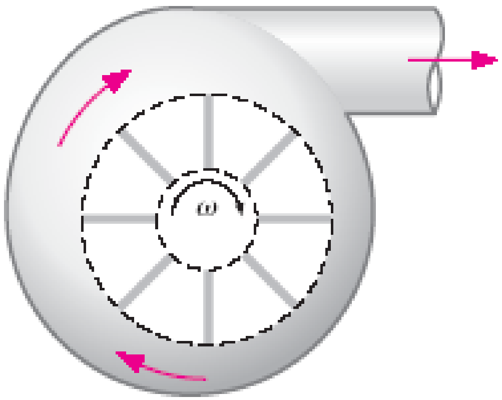
Most of the centrifugal pumps have backward-inclined blades, however also radial (straight) and forward-inclined blades can be found in practice.

Backward-inclined blades



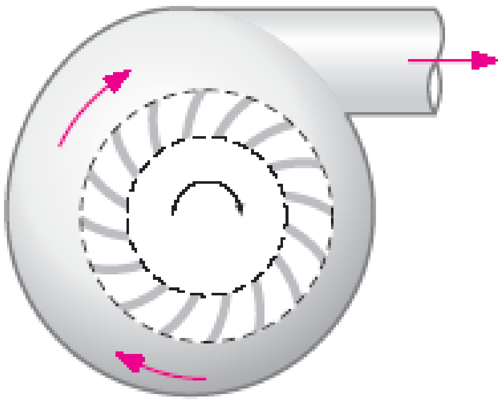
Most common and efficient;
Intermediate pressure rise;
Less robust.

Radial (straight) blades



Simplest geometry;
High pressure rise over a range of flow rates;
Less robust.

Forward-inclined blades



More blades but smaller;
Lowest pressure rise per unit of input power;
Lowest efficiency;
More robust.

Integral analysis of centrifugal pumps

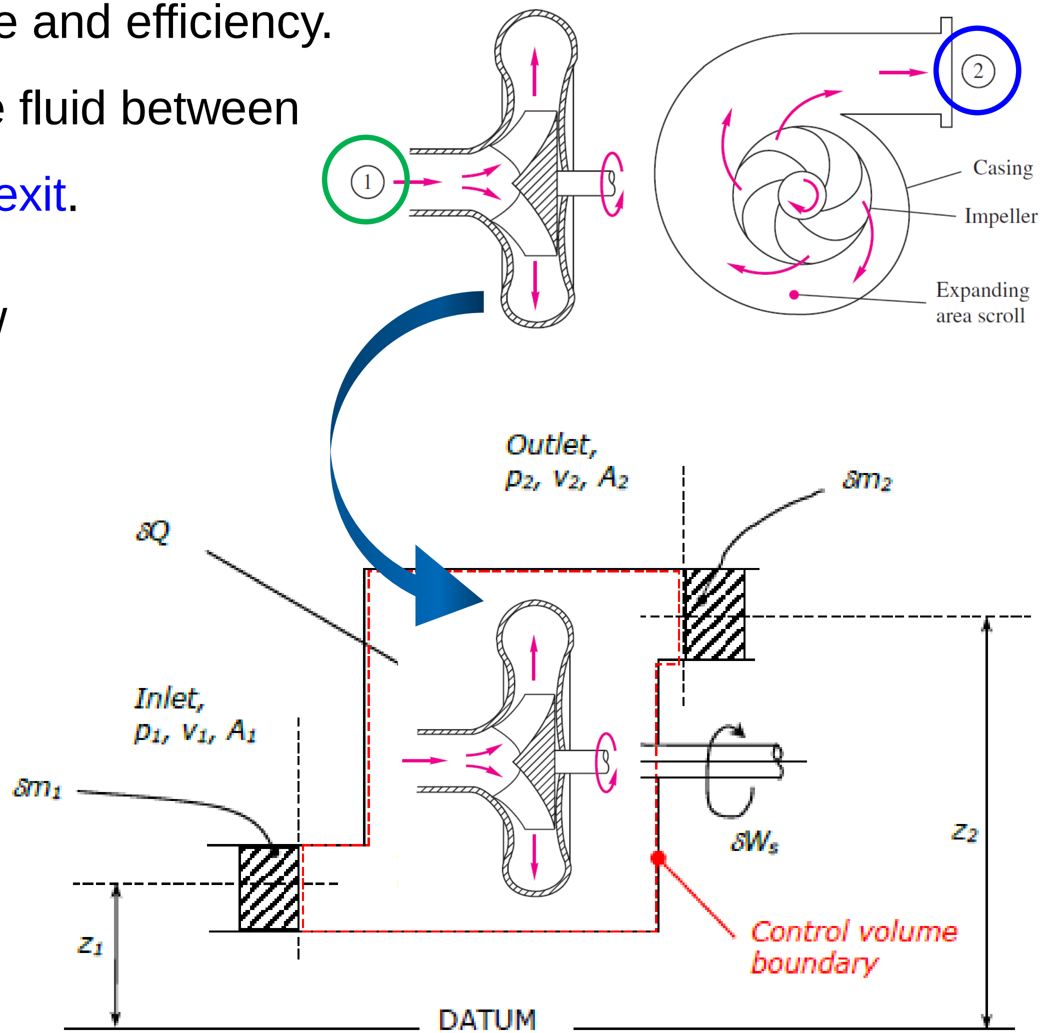
Now we study centrifugal pumps more in detail, in order to derive mathematical expressions for the pump performance and efficiency.

The pump increases the energy of the fluid between section 1, the eye, and section 2, the exit.

In TF1 (pag. 109), you have seen how to write the steady flow energy equation (SFEE) for an open system, between an inlet and outlet section:

$$q + w_s = e_2 - e_1 + \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}$$

It's a specific energy balance, i.e. energy per unit mass [J/kg=m^2/s^2]!



TF1 notes, p. 107

Integral analysis of centrifugal pumps

$$q + w_s = e_2 - e_1 + \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}$$

It's a specific energy balance, i.e.

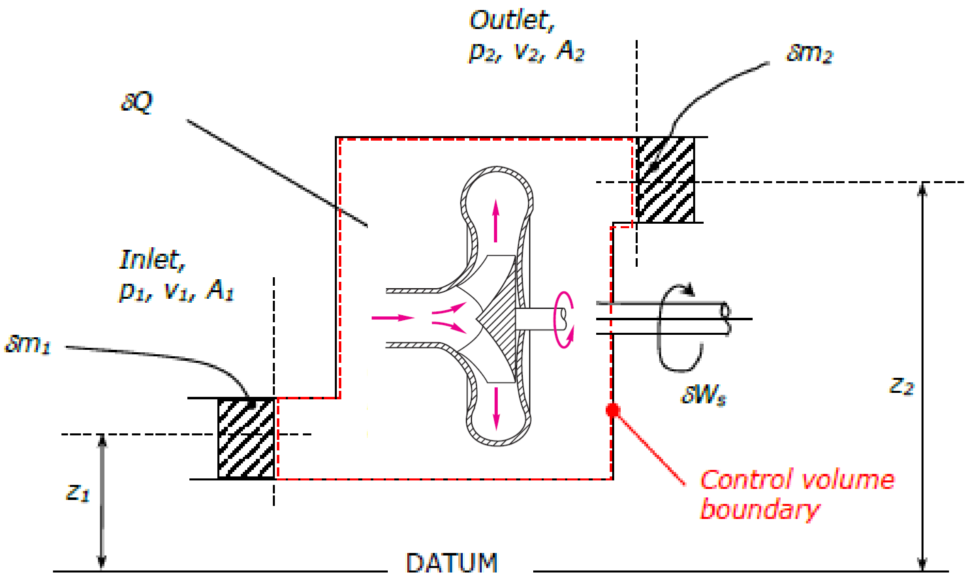
energy per unit mass [J/kg=m^2/s^2]!

q: heat given to the system

w_s : work supplied to the shaft

$e = u + \frac{v^2}{2} + gz$ Specific energy of the fluid

Internal energy (u), Kinetic energy ($\frac{v^2}{2}$), Potential energy (gz)



$\frac{p}{\rho}$: Work done by pressure forces

Incompressible flow: $\rho_1 = \rho_2$

$$\Rightarrow \quad q + w_s = \left[u_2 + \frac{p_2}{\rho} + gz_2 + \frac{v_2^2}{2}\right] - \left[u_1 + \frac{p_1}{\rho} + gz_1 + \frac{v_1^2}{2}\right]$$

Integral analysis of centrifugal pumps

$$q + w_s = \left[u_2 + \frac{p_2}{\rho} + gz_2 + \frac{v_2^2}{2}\right] - \left[u_1 + \frac{p_1}{\rho} + gz_1 + \frac{v_1^2}{2}\right]$$

Rearranging:

$$\frac{w_s}{g} = \left[\frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g}\right] - \left[\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g}\right] + \left[\frac{u_2 - u_1 - q}{g}\right]$$

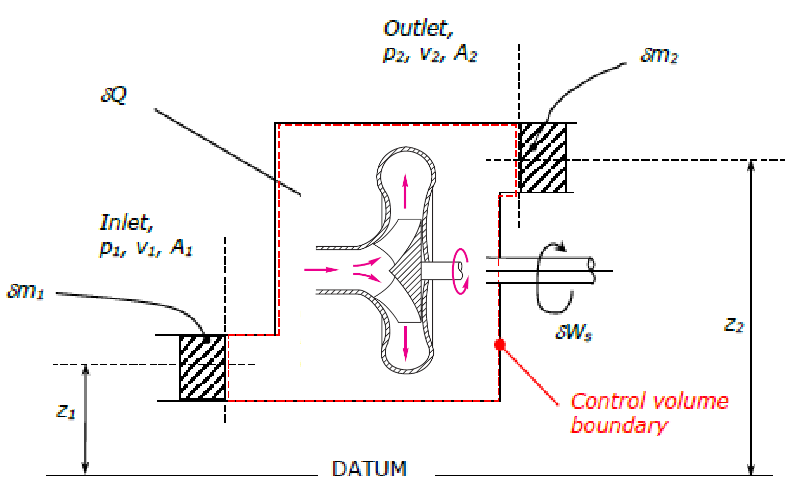
$\frac{w_s}{g}$: Head supplied to the pump [units: m], we will call it H_s

$\left[\frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g}\right]$: Total head of fluid at exit, we will call it $H_{T,2}$

$\left[\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g}\right]$: Total head of fluid at inlet, $H_{T,1}$

$\left[\frac{u_2 - u_1 - q}{g}\right]$: Friction head losses, H_f

$$H_s - H_f = H_{T,2} - H_{T,1}$$



Integral analysis of centrifugal pumps

$$H_s - H_f = H_{T,2} - H_{T,1}$$

The head supplied to the pump, minus the friction loss, is equal to the increase of total head of the fluid

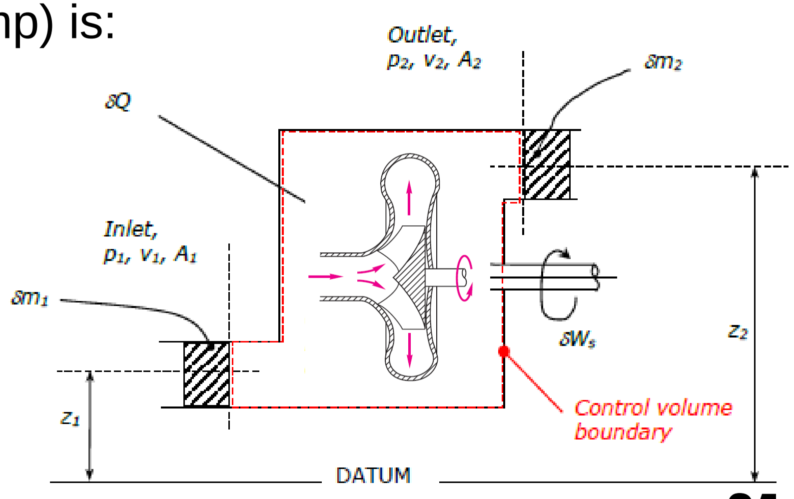
$$H = H_{T,2} - H_{T,1} = \left[\frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g}\right] - \left[\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g}\right]$$

But:

- The difference in elevation between inlet and outlet is usually negligible, $z_1 \approx z_2$
- The inlet and outlet diameter (or area) are usually the same, so that by continuity $v_1 \approx v_2$

Therefore, the net pump head (output of the pump) is:

$$H \approx \frac{p_2 - p_1}{\rho g}$$



And the power delivered to the fluid is:

$$P_w = \rho Q g H$$

Units: Watt, W

Q: volumetric flow rate, m^3/s

Integral analysis of centrifugal pumps

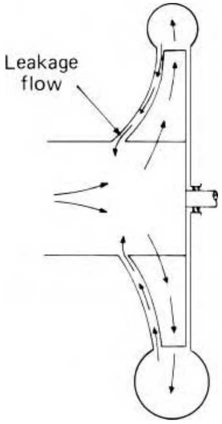
Power delivered to the fluid: $P_w = \rho Q g H$ Also called **water horsepower**

The power actually supplied to the pump (brake horsepower) is: $P = \omega T$, with ω shaft angular velocity [rad/s] and T torque [N·m]. This leads to the overall pump efficiency:

$$\eta = \frac{P_w}{P} = \frac{\rho Q g H}{\omega T}$$

The efficiency is composed of three parts:

- Hydraulic efficiency $\eta_h = 1 - \frac{H_f}{H_s}$ Where H_f incorporates losses due to imperfect match at inlet/outlet and friction
- Mechanical efficiency $\eta_m = 1 - \frac{P_f}{P}$ Where P_f accounts for power loss due to mechanical friction in the bearings and other contact points in the machine
- Volumetric efficiency $\eta_v = \frac{Q}{Q + Q_L}$ Where Q_L is the loss due to leakage flow in the impeller casing clearances

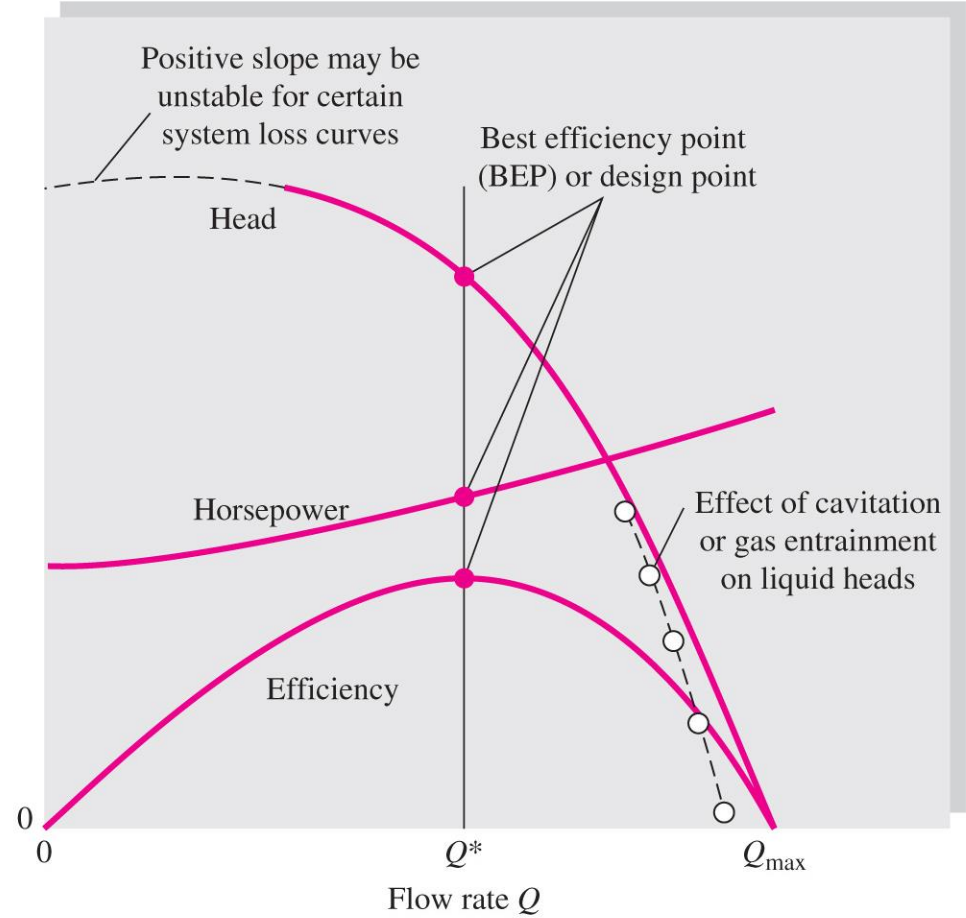


$$\eta = \eta_h \eta_m \eta_v$$

Centrifugal pump performance

Centrifugal pumps manufacturers provide performance curves obtained via experimental tests.

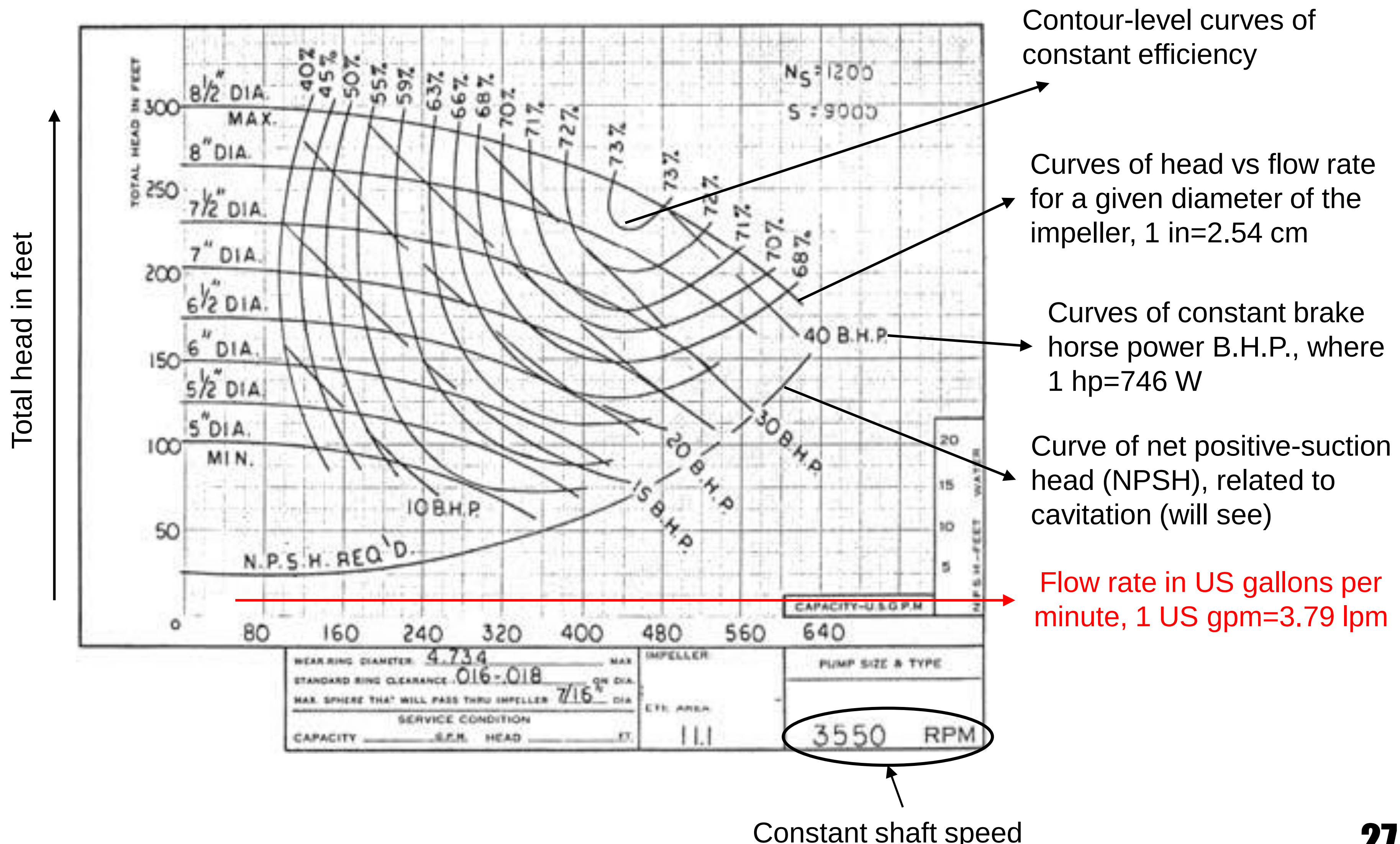
Pump head H, brake horsepower P and efficiency η are given as a function of discharge Q. The shaft speed ω and impeller size are constant.



- At low Q, H is large but the pump is unstable (surging).
- H decreases as Q increases, down to $H = 0$ when $Q = Q_{max}$.
- The power P necessary to run the pump increases monotonically with Q.
- The efficiency η is zero when $Q = 0$ and $Q = Q_{max}$, and is max at $Q^* \approx 0.6Q_{max}$. This identifies the best efficiency point BEP, at which $H = H^*$ and $P = P^*$. Typically, $\eta_{max} = 0.8 - 0.9$.

Centrifugal pump performance

Performance curves from manufacturers typically look like this



Worked example 1

An 8 inch pump of the type described by the performance curves below is required to pump water at 0.025 m³/s. What is the efficiency and pump power for this operation point? What differential pressure does the pump generate?

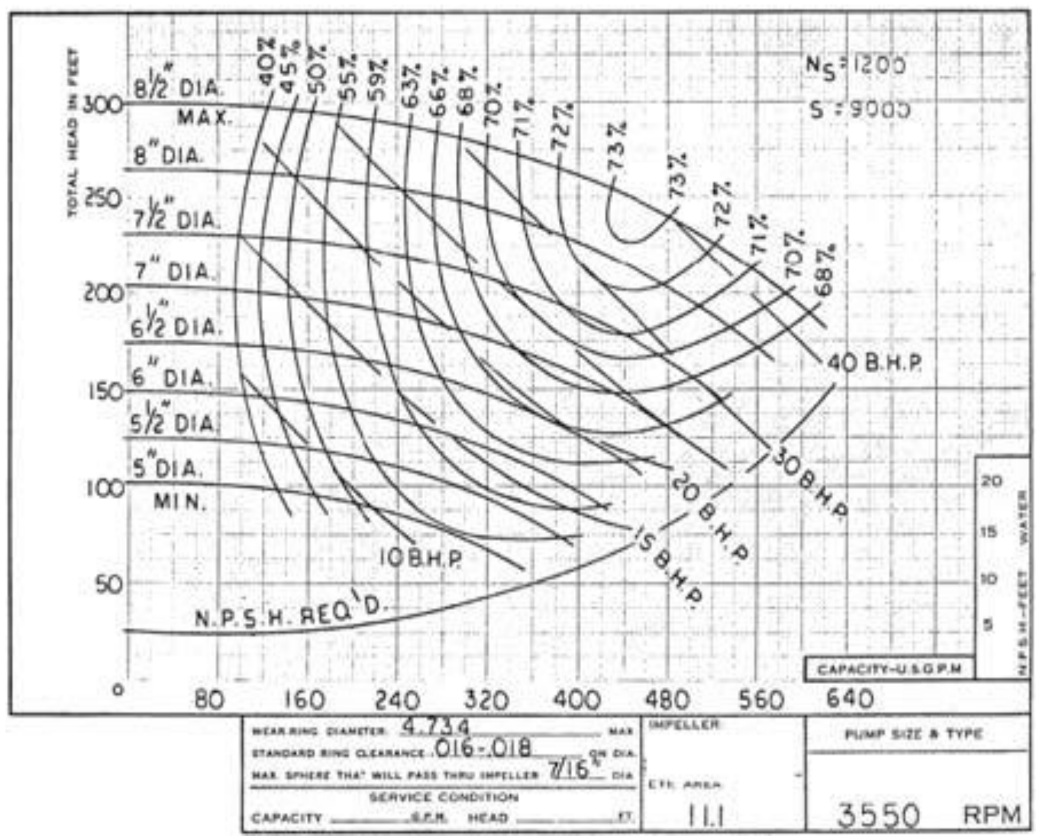
Remember the conversion rates:

1 US gpm = 3.79 lpm

1 litre = 10^{-3} m^3

1''= 0.0254 m

1 foot = 12''=0.3048 m



Worked example 1

Solution:

$$Q = 0.025\ \frac{m^3}{s} = \frac{0.025}{3.79 \times 10^{-3}} \times 60 = 396\ US\ gpm$$

Intersection with 8'' curve yields efficiency of (little above) $\eta \approx 0.72$.

The brake horse power (required to run the pump) is (little above)

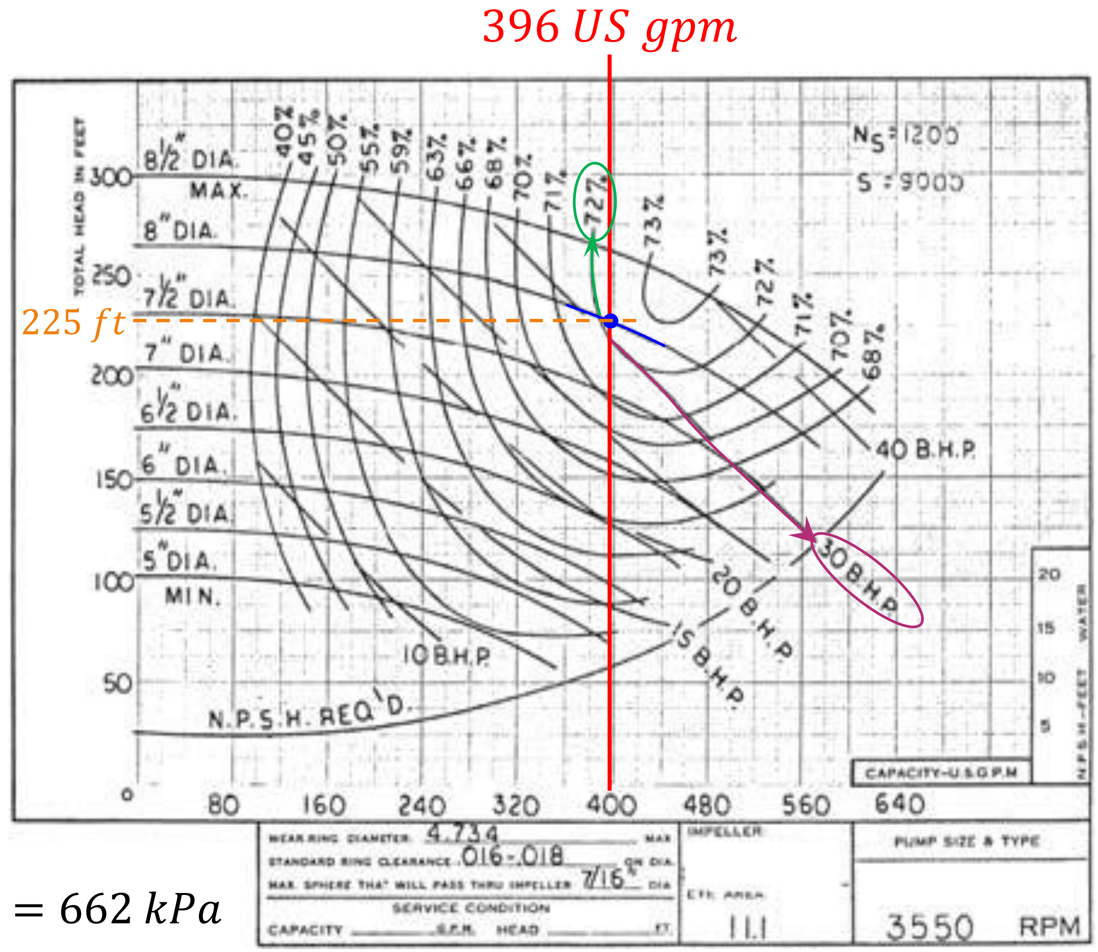
$P \approx 30\ bhp = 30 \times 746 = 22.4\ kW$.

The generated total head can be read by intercepting the y-axis:

$H = 225\ ft = 225 \times 0.3 = 67.5\ m$.

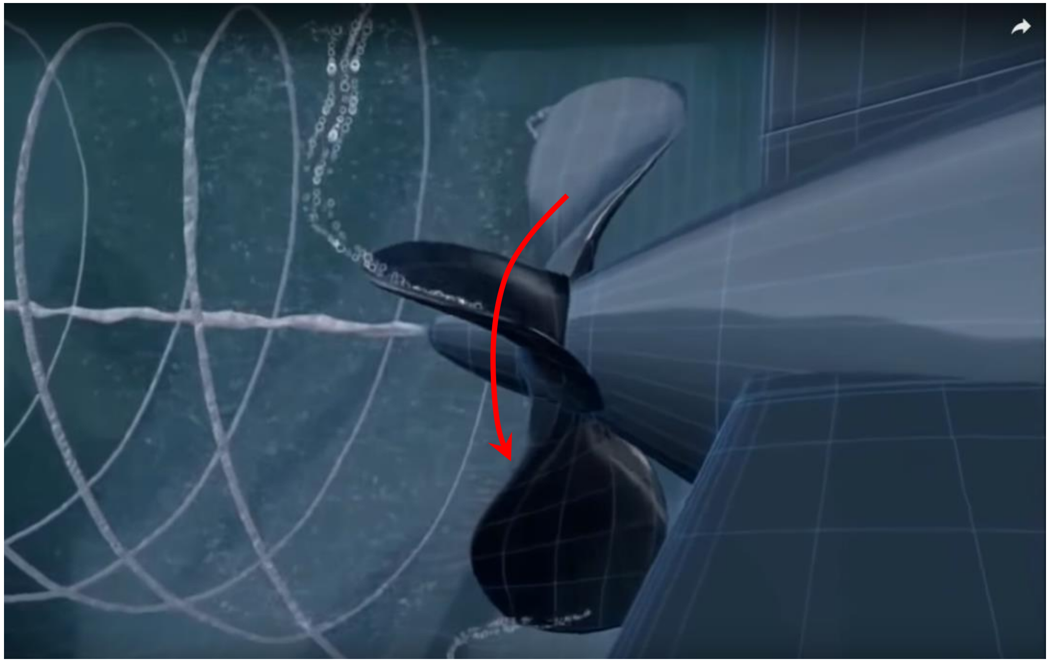
And the differential pressure:

$$\Delta p = \rho g H = 1000 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2} \cdot 67.5\ m = 662\ kPa$$



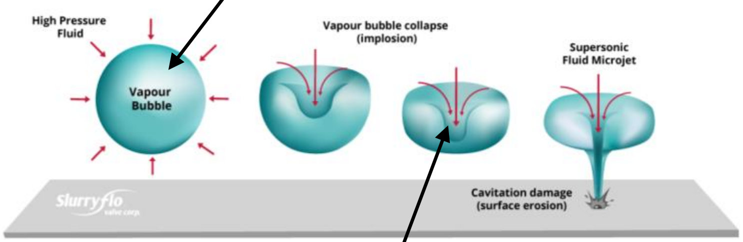
Cavitation

Cavitation is the generation of vapor bubbles by a sudden drop in pressure, followed by their implosion when the original pressure is restored. Their implosion generates a high-pressure wave that can damage (i.e. form cavities) nearby solid surfaces.



https://www.youtube.com/watch?v=ON_irzFAU9c

Vapor bubble generated by a previous low pressure situation



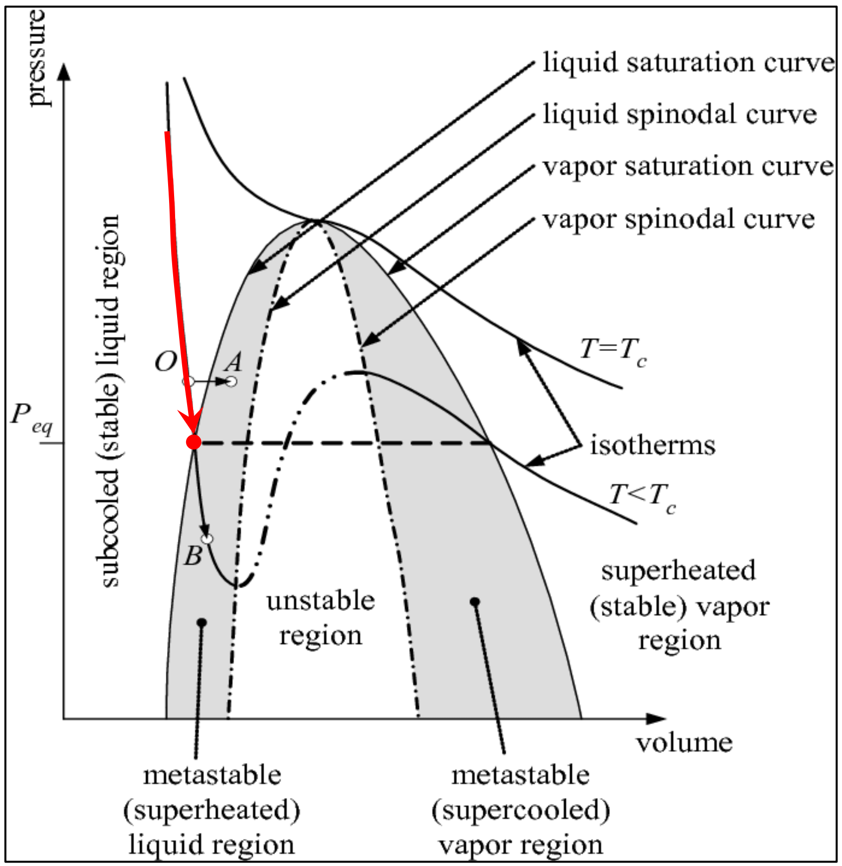
https://www.slurryflo.com/cavitation

Bubble collapse due to restored (high) pressure

Cavitation

Why does a drop in pressure generate vapor bubbles?

At the liquid state, we move along an isotherm line towards lower pressures. When the line crosses the liquid saturation curve, vapor is created.



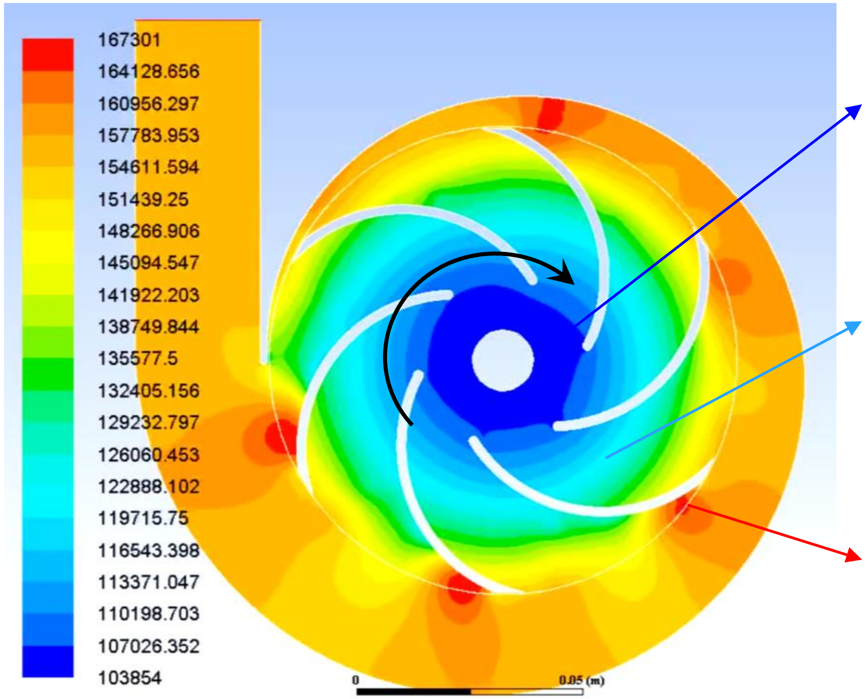
Typical pressure-volume phase diagram for a pure fluid

Example: you can boil water at 20 °C by decreasing its pressure to 0.0231 atm.

And why/where does the pressure drop in a pump?

Cavitation

And why/where does the pressure drop in a pump?

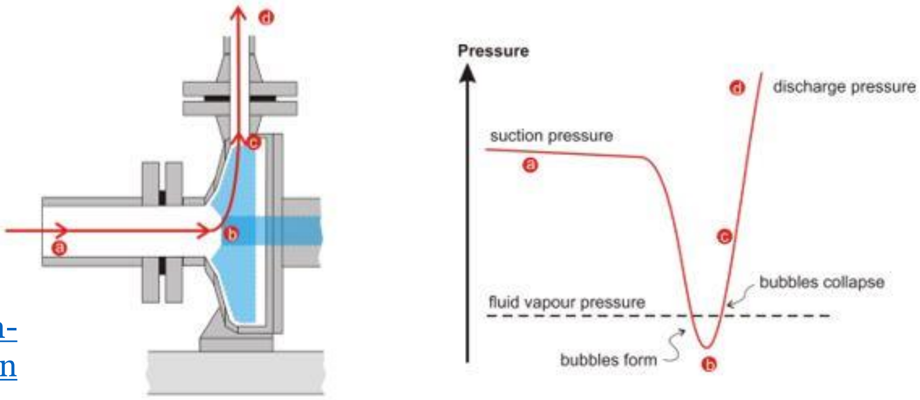


The fluid pressure drops at the impeller's eye, where it has the minimum value; if pressure falls below the saturation pressure, bubbles appear

Pressure grows as the fluid flows between the blades, because the ducts are diverging

Pressure is maximum at the trailing edge of the blades, on their front side; here is where cavitation occurs: bubbles implode and the generated pressure wave creates wear on the blades.

https://www.youtube.com/watch?v=g105Z907b0

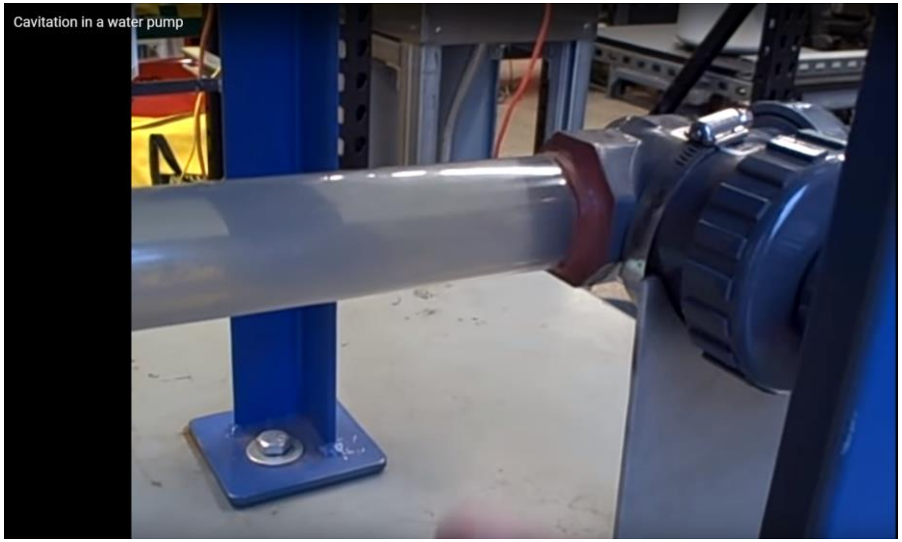


https://www.michael-smith-engineers.co.uk/resources/useful-info/pump-cavitation

Cavitation

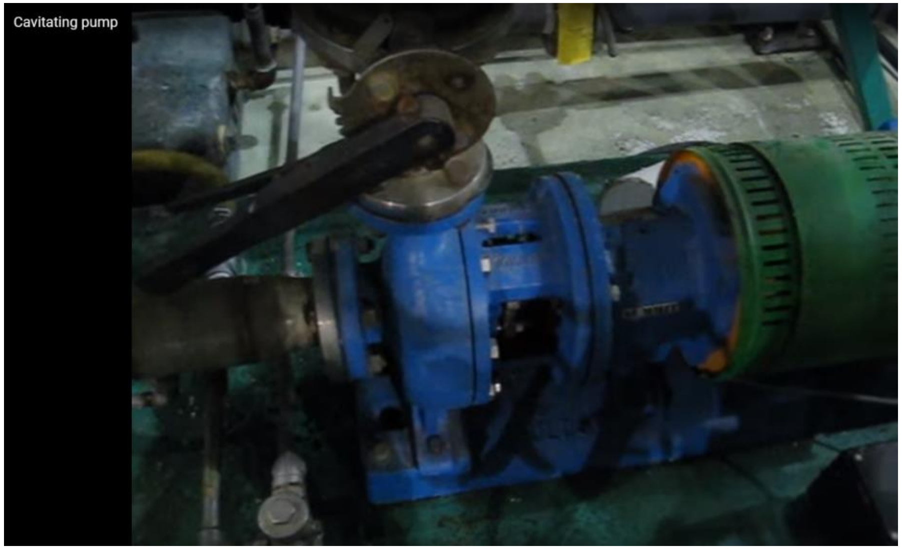
https://www.youtube.com/watch?v=eMDAwoTXvUo&feature=emb_logo

How does cavitation look like…



https://www.youtube.com/watch?time_continue=1&v=1Lbxtjfdat4&feature=emb_logo

…and how does it sound



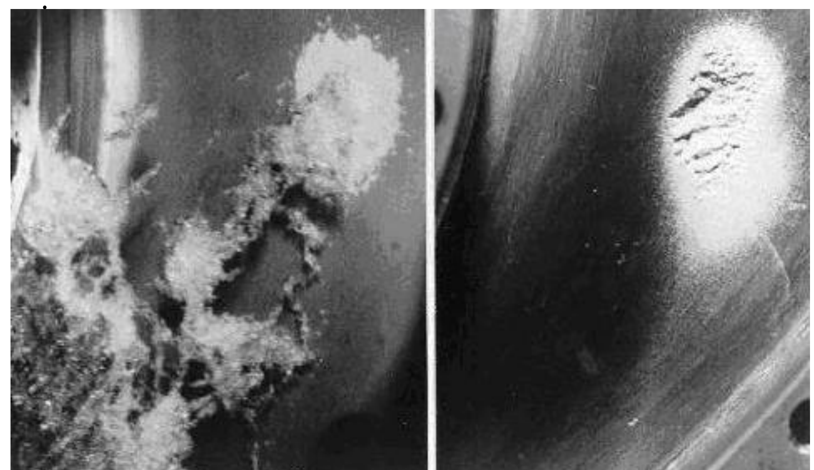
University of Nottingham
UK | CHINA | MALAYSIA

Blades wear due to cavitation

http://processindustryforum.com/wp-content/uploads/2014/03/Cavitation-damage-pump-impeller.jpg

http://cdn.powermag.com/wp-content/uploads/2014/03/PWR_030114_OMCoatings_fig6

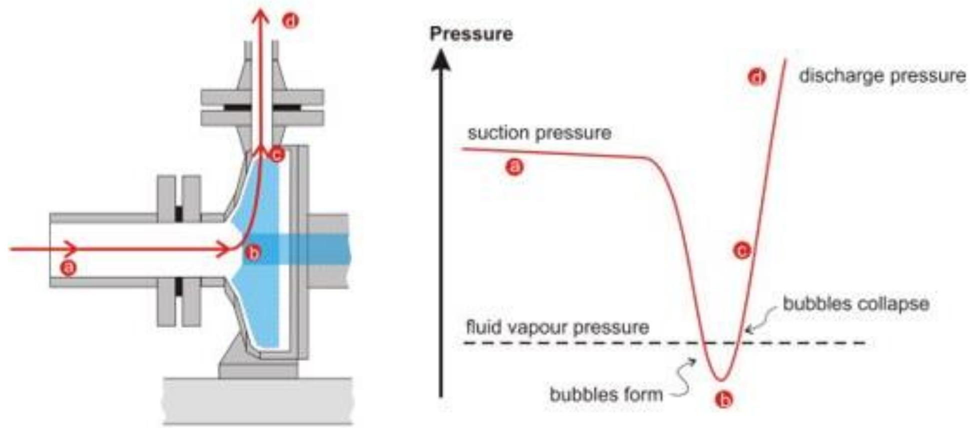
http://www.marin.nl/upload/ef906540-37a5-4c5a-8e49-d9bf546a1366_1228144642562-erosion.JPG



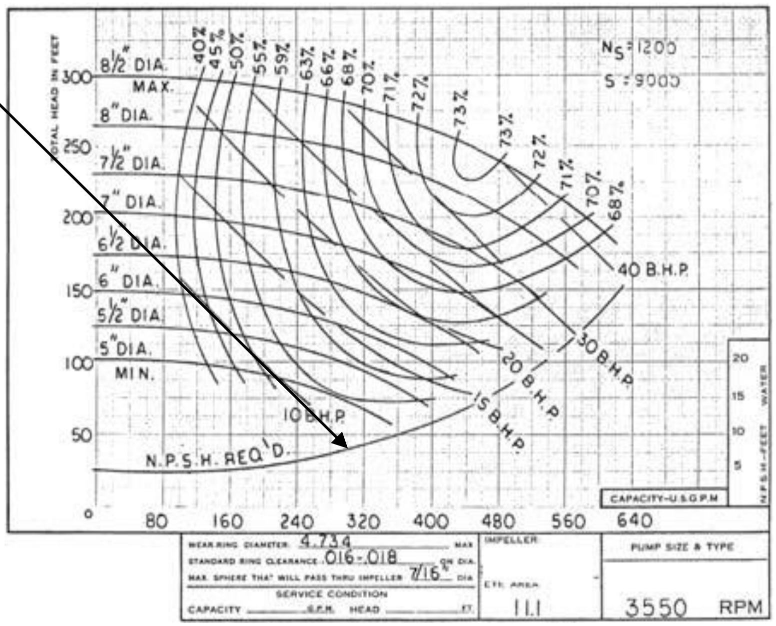
http://www.marin.nl/upload/ef906540-37a5-4c5a-8e49-d9bf546a1366_1228144642562-erosion.JPG

Net Positive-Suction Head (NPSH)

The manufacturer provides the head loss through the impeller eye, then we must make sure that the pressure of the fluid at the inlet is large enough to prevent it from falling below the saturation pressure at that fluid temperature.



The head loss is called ***Net Positive-Suction Head (NPSH)***, and is the head required at the inlet to avoid cavitation. We need to make sure that the following condition is satisfied:



$$H_i - \frac{p_v}{\rho g} > NPSH$$

$$H_i = \frac{p_i}{\rho g} + \frac{v_i^2}{2g}$$ Total head at the inlet

$$p_v = p_{sat}(T_i)$$ Fluid saturation pressure at T_i

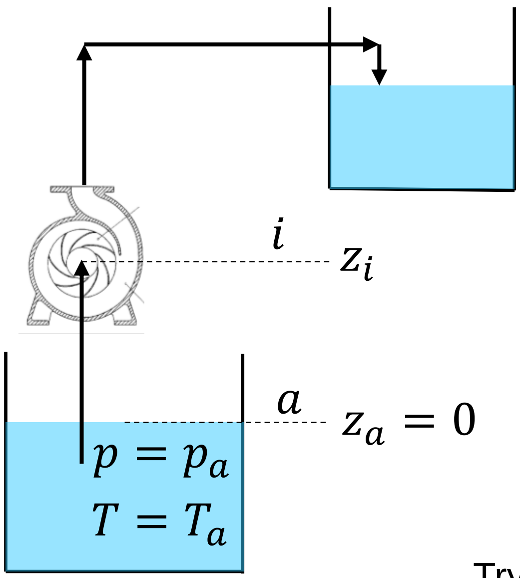
Net Positive-Suction Head (NPSH)

We need to make sure that the following condition is satisfied:

$$\frac{p_i}{\rho g} + \frac{v_i^2}{2g} - \frac{p_v}{\rho g} > NPSH$$

So, it is important that the pressure at the inlet is as large as possible. What can we do in practice? Usually the fluid comes from a reservoir, where $v = 0$.

We apply the *extended Bernoulli equation* (TF1, p. 116) between a and i:



$$H_i = H_a - H_f - z_i$$

H_f: friction head loss between reservoir and pump inlet

$$H_a = \frac{p_a}{\rho g}$$

$$\frac{p_i}{\rho g} + \frac{v_i^2}{2g} = \frac{p_a}{\rho g} - H_f - z_i$$

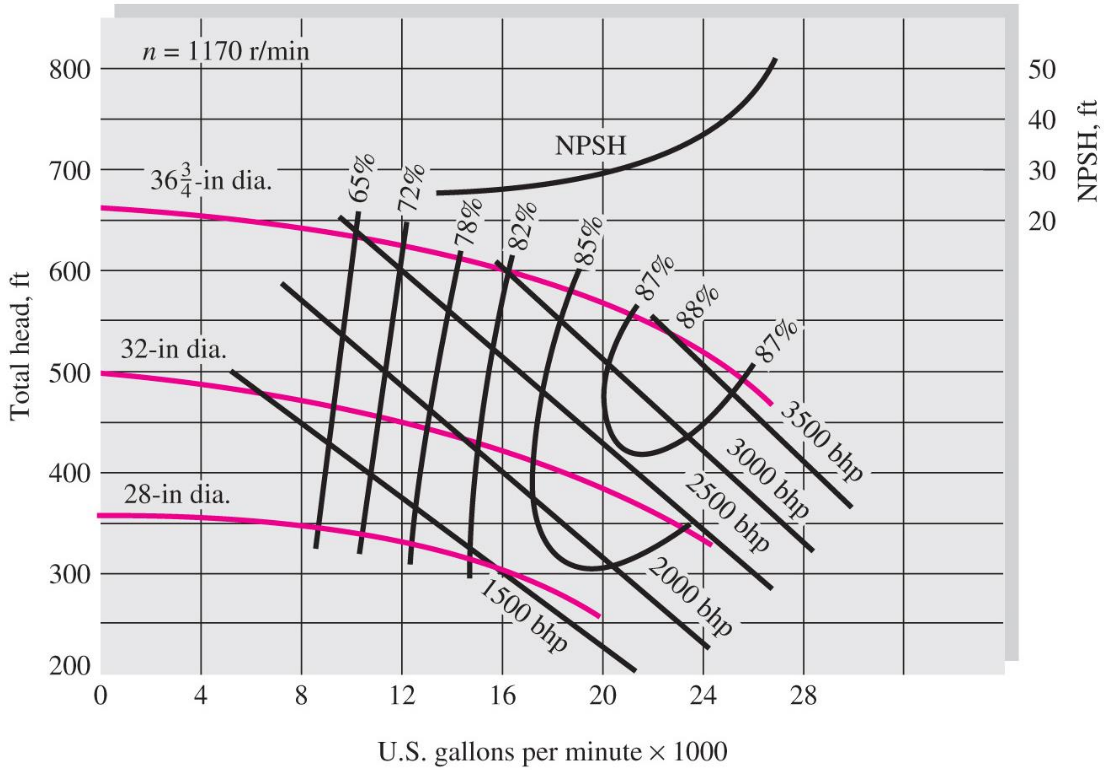
$$\frac{p_a}{\rho g} - H_f - z_i - \frac{p_v}{\rho g} > NPSH$$

Try to reduce friction losses, e.g. shorter/smoother pipes

Install the pump at low level, even below that of the reservoir so that $z_i < 0$

Worked example 2

The performance data for a 32” pump is given below. The pump is to pump 24,000 US gpm of water from a reservoir where the pressure at the surface is 1.01 bar. If the head loss due to friction from reservoir to pump inlet is 6 ft, how far below the reservoir surface should the pump inlet be placed to avoid cavitation for water at 15.5°C, density 1000 kg/m^3 and p_v= 1.8 kPa.



Worked example 2

Solution

From the chart,

NPSH=37 ft=11.28 m.

From the extended Bernoulli equation,

$$\frac{p_a}{\rho g} - H_f - z_i - \frac{p_v}{\rho g} > NPSH$$

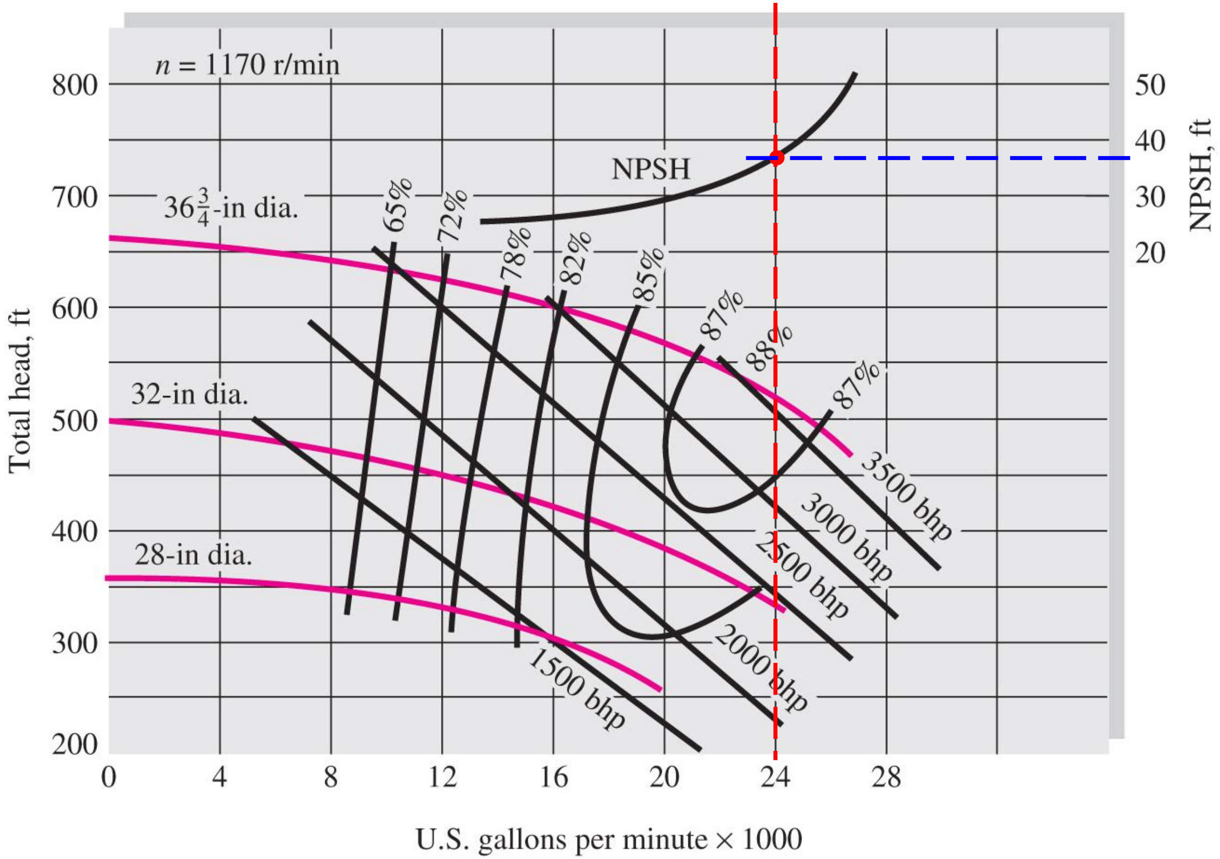
where H_f is given, $H_f = 6\ ft = 1.83\ m$. Therefore,

$$z_i < \frac{p_a}{\rho g} - H_f - \frac{p_v}{\rho g} - NPSH$$

$$\frac{p_a}{\rho g} = \frac{1.01 \times 10^5 Pa}{1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2}} = 10.29\ m$$

$$\frac{p_v}{\rho g} = \frac{1.8 \times 10^3 Pa}{1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2}} = 0.18\ m$$

$24000\ US\ gpm$



$$z_i < 10.29 - 1.83 - 0.18 - 11.28 = -3\ m$$

The pump inlet should be at least 3 m under the reservoir level

Dimensionless pump performance

Dimensional analysis can be applied to pumps, in order to obtain the relevant nondimensional groups that characterise the pump performance.

- Dependent (output) variables: pump head H, brake horsepower P.
- Independent (input) variables: discharge Q, impeller diameter D, shaft speed n, fluid density ρ and viscosity μ, surface roughness ϵ.
- The dimensional relationships we are looking for are:

$$gH = f_1(Q, D, n, \rho, \mu, \epsilon), \qquad P = f_2(Q, D, n, \rho, \mu, \epsilon)$$

Note that we choose gH instead of H for dimensional reasons.

- 8 variables, 3 dimensions, therefore we expect 5 non-dimensional groups.
- Using the pi-theorem, with repeating variables ρ, n, D and non-repeating variables gH, P, Q, μ, ϵ, we obtain:

$$\Pi_1 = \frac{gH}{n^2 D^2}$$ Head coefficient C_H

$$\Pi_2 = \frac{P}{\rho n^3 D^5}$$ Power coefficient C_P

$$\Pi_3 = \frac{Q}{nD^3}$$ Capacity coefficient C_Q

$$\Pi_4 = \frac{\rho n D^2}{\mu}$$ Reynolds number Re

$$\Pi_5 = \frac{\epsilon}{D}$$ Roughness parameter

Dimensionless pump performance

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$$\Pi_4 = \frac{\rho n D^2}{\mu}$$ Reynolds number Re

$$\Pi_5 = \frac{\epsilon}{D}$$ Roughness parameter

Therefore, we find that:

$$C_H = g_1\left(C_Q, Re, \frac{\epsilon}{D}\right), \qquad C_P = g_2\left(C_Q, Re, \frac{\epsilon}{D}\right)$$

For pumps, it is usually assumed that the Reynolds number and roughness parameter are constant for a set of similar pumps, and therefore:

$$C_H \approx g_3(C_Q), \qquad C_P \approx g_4(C_Q)$$

The pump efficiency is already dimensionless and related to the other groups as:

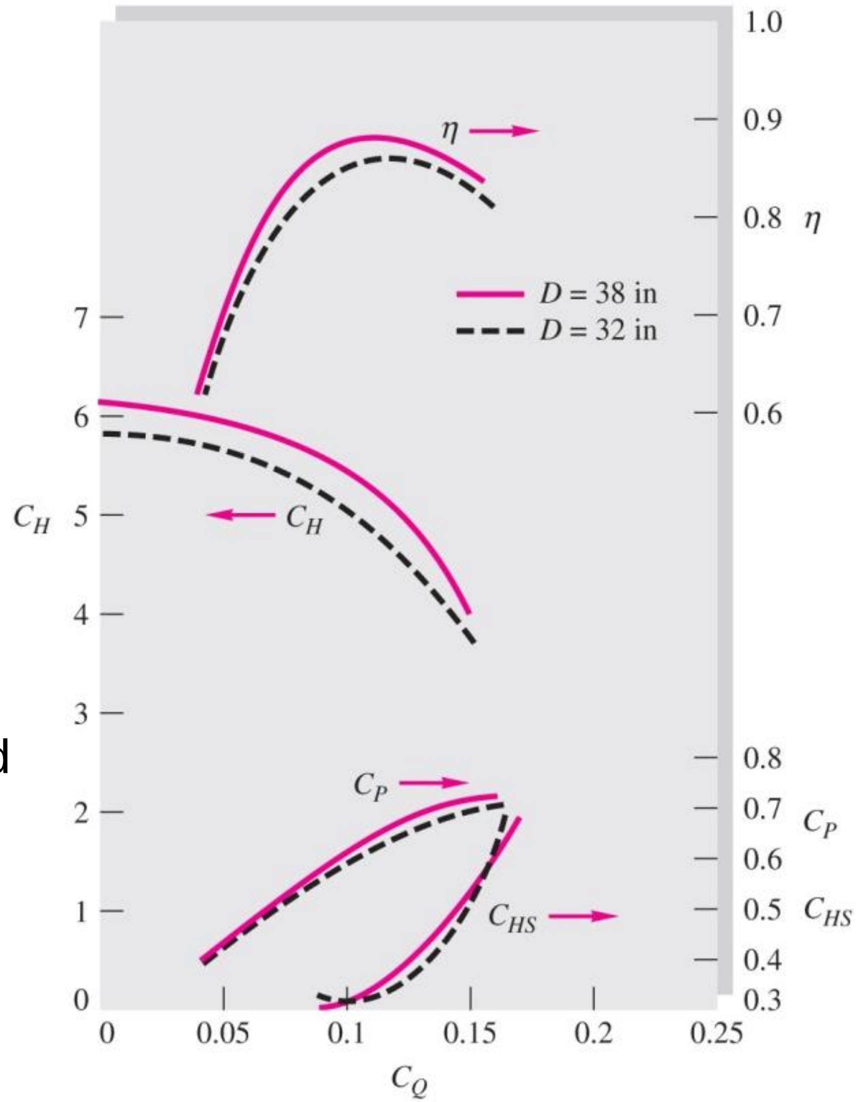
$$\eta = \frac{\rho Q g H}{P} = \frac{\rho Q g H}{P} \cdot \frac{nD^3}{nD^3} \cdot \frac{n^2D^2}{n^2D^2} \cdot \frac{\rho n^3 D^5}{\rho n^3 D^5} = \frac{C_H C_Q}{C_P} \Rightarrow \eta \approx \eta(C_Q)$$

We can also define a nondimensional NPSH: $C_{HS} = \dfrac{g \cdot NPSH}{n^2 D^2} \approx C_{HS}(C_Q)$

Dimensionless pump performance

Nondimensional plot of pump performance data for two pumps of the same family, with different impeller diameters.

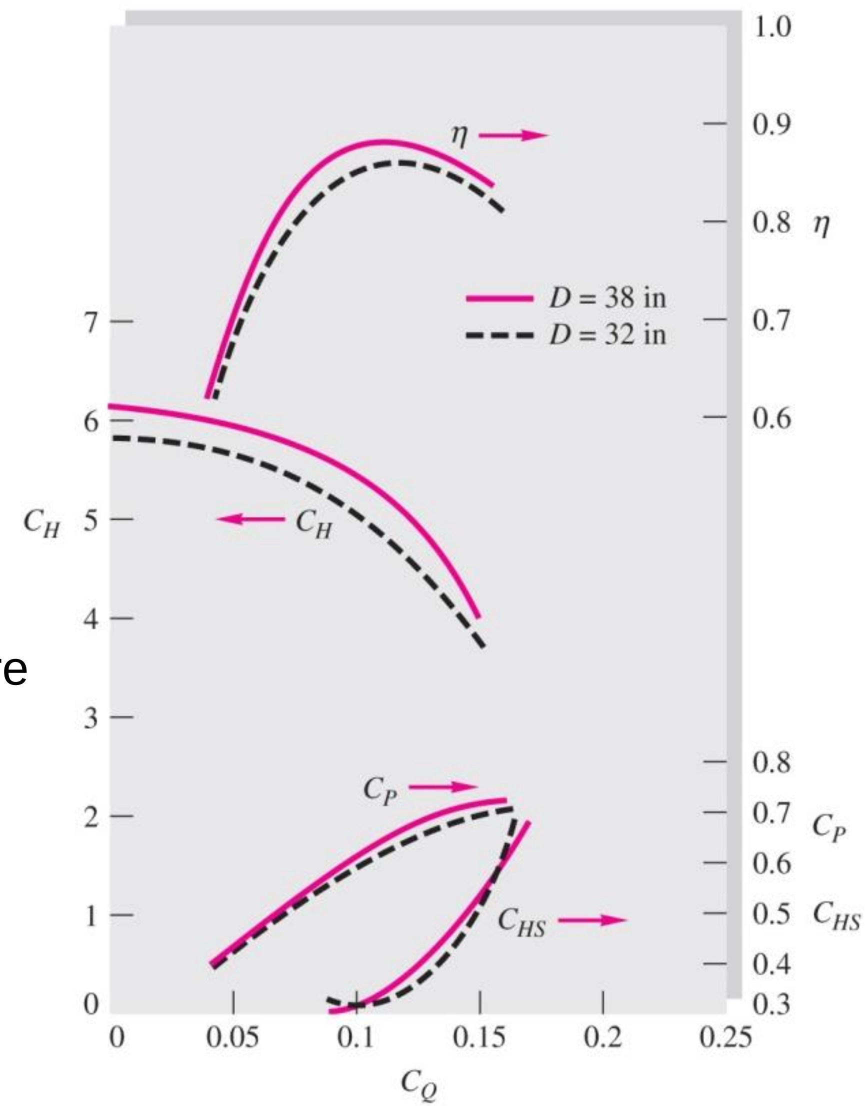
- The curves for different D almost overlap, such that C_H, C_P, C_{HS}, η depend (almost) only on C_Q, as suggested by the dimensional analysis.
- This validates the results of the dimensional analysis.
- The larger pump exhibits slightly larger η and C_H: this because, being larger, it has smaller roughness parameter, smaller clearance ratios, and larger Re; hence, it develops slightly more head and is more efficient.



Worked example 3

A pump from the family of the figure has a diameter of 21" and operates at 1500 rpm. Estimate the discharge and differential pressure for this pump when operating at BEP (best efficiency point) for water with density 1000 kg/m^3. What is the input power required for this pump?

Note: the manufacturer states that the curves are obtained for n given in revolutions per second.



Worked example 3

Solution

For BEP, consider $C_{Q^*} = 0.118$, $\eta = 0.87$, $C_{H^*} = 4.7$ and $C_{P^*} = 0.63$.

Discharge:

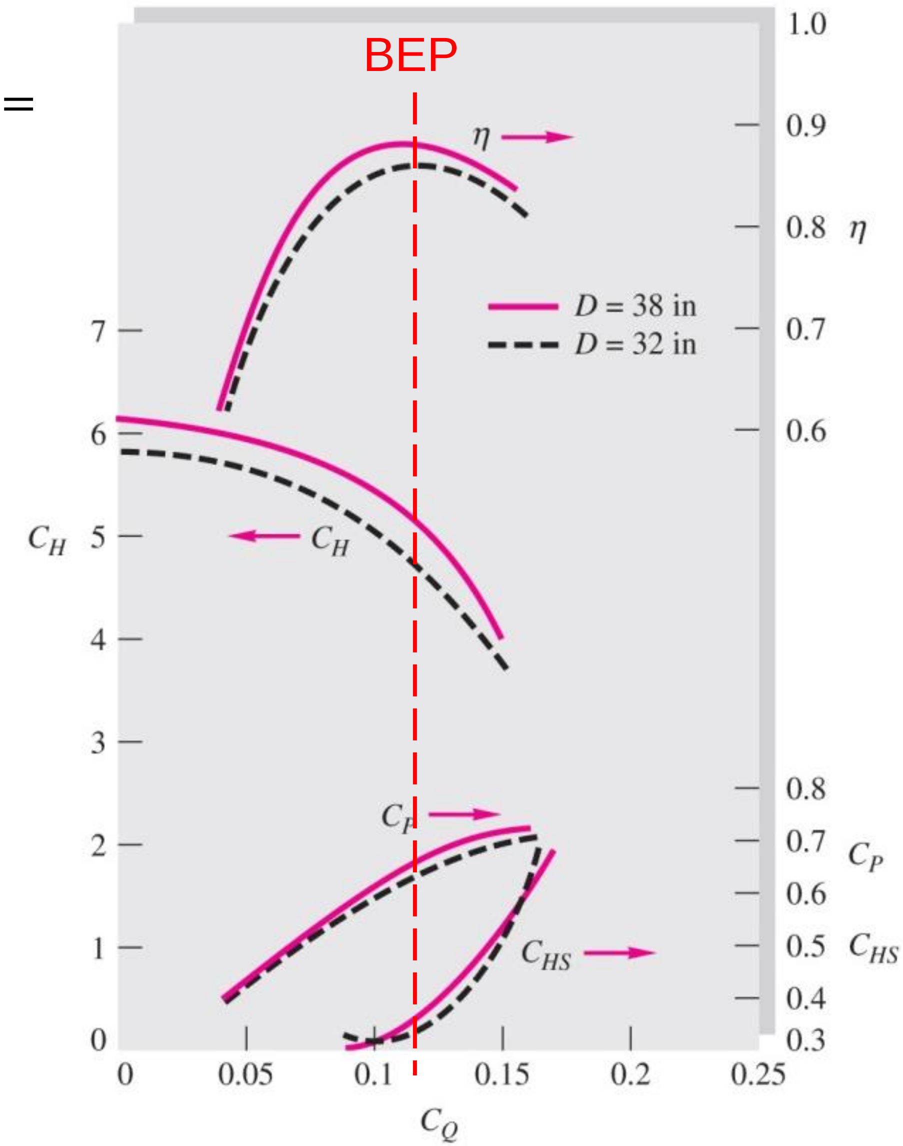
$$C_{Q^*} = \frac{Q^*}{nD^3} \Longrightarrow Q^* = C_{Q^*} n D^3$$

with $n = 1500\ rpm = 25\ rps$, $D = 21'' = 0.533\ m$.

$$Q^* = C_{Q^*} n D^3 = 0.118 \times 25\ rps \times (0.533\ m)^3$$
$$= 0.45\ \frac{m^3}{s}$$

Total head:

$$C_{H^*} = \frac{gH}{n^2 D^2} \Longrightarrow H^* = \frac{C_{H^*} n^2 D^2}{g}$$
$$= \frac{4.7 \times (25\ rps)^2 \times (0.533\ m)^2}{9.81\ \ m/s^2} = 85.1\ m$$



Worked example 3

Total head:

$$C_{H^*} = \frac{gH^*}{n^2D^2} \Longrightarrow H^* = \frac{C_{H^*}n^2D^2}{g}$$

$$= \frac{4.7 \times (25\,rps)^2 \times (0.533\,m)^2}{9.81\ m/s^2} = 85.1\,m$$

Differential pressure:

$$\Delta p = \rho g H^* = 1000\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 85.1\,m$$

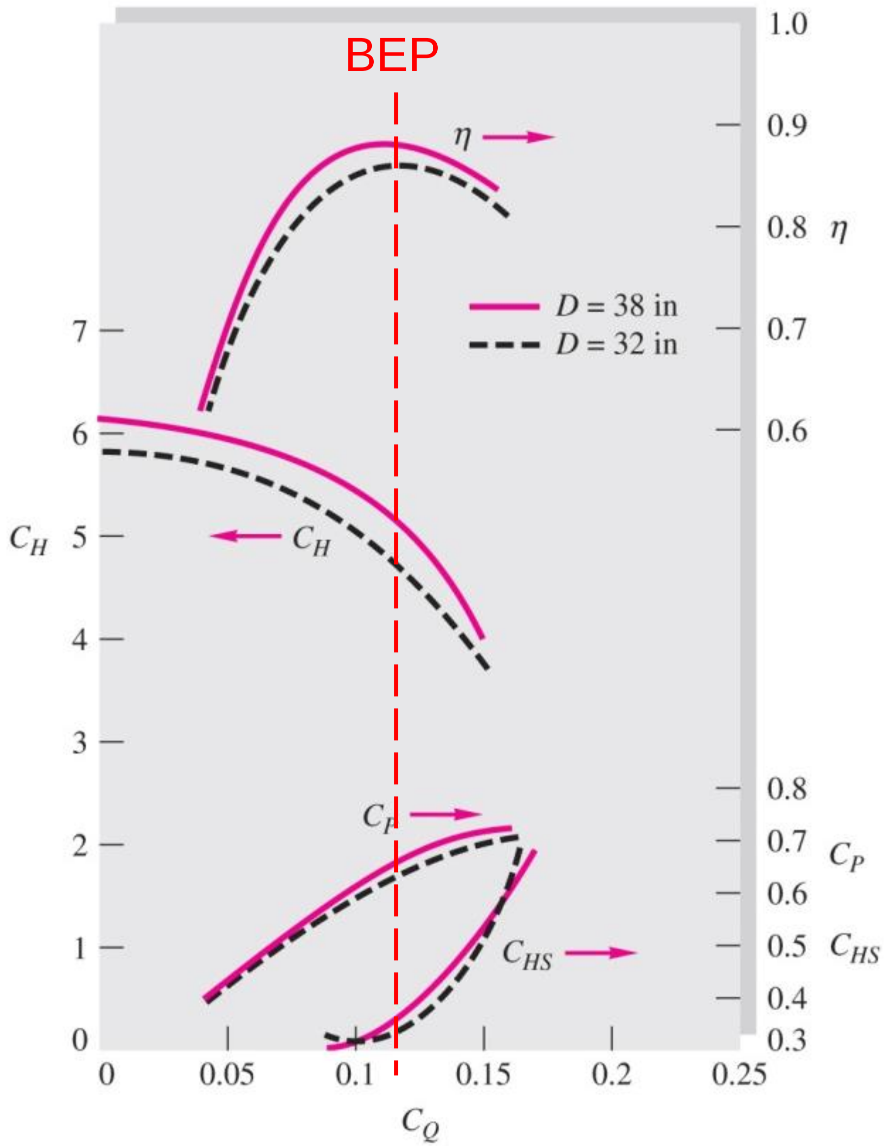
$$= 835\,kPa$$

Power input:

$$C_{P^*} = \frac{P^*}{\rho n^3 D^5} \Longrightarrow P^* = C_{P^*}\rho n^3 D^5$$

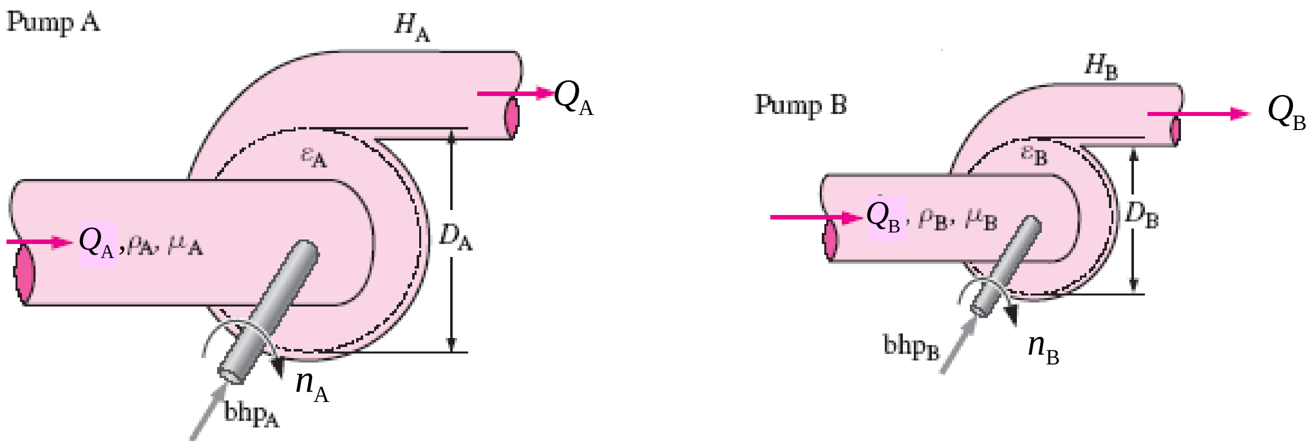
$$= 0.63 \times 1000\frac{kg}{m^3} \times (25\,rps)^3 \times (0.533\,m)^5$$

$$= 423\,kW$$



Similarity rules for pumps

Dimensional analysis suggests that two pumps of a family of similar pumps, when operated at homologous points (which means $C_{Q,1} = C_{Q,2}$), they will have same head and power coefficient, $C_{H,1} = C_{H,2}$ and $C_{P,1} = C_{P,2}$.



This enables us to find relationships between their flow rates, heads and powers.

Let's consider pump 1 and pump 2 of the same geometric family:

$$C_{Q,1} = C_{Q,2} \Longrightarrow \frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3} \Longrightarrow \boxed{\frac{Q_2}{Q_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3}$$

Similarity rules for pumps

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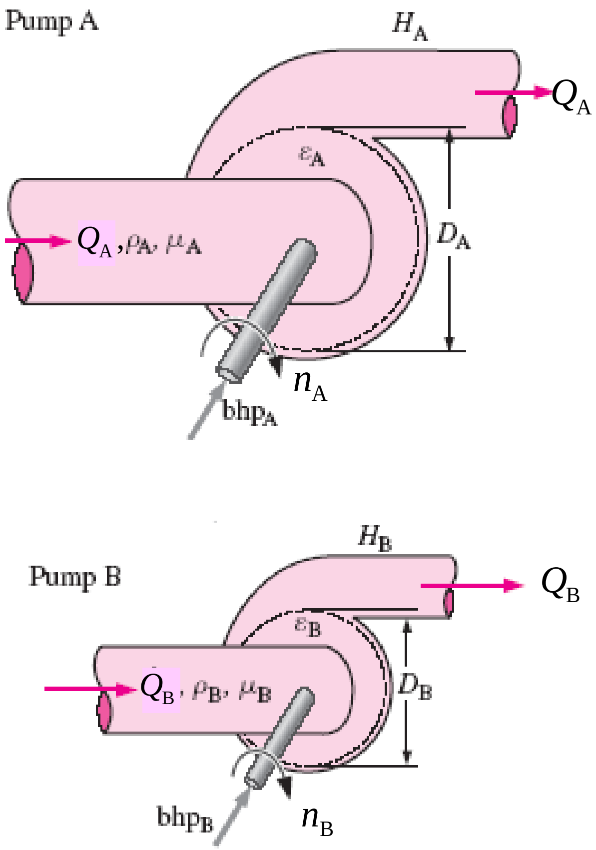
$$C_{H,1} = C_{H,2} \Rightarrow \frac{gH_1}{n_1^2 D_1^2} = \frac{gH_2}{n_2^2 D_2^2} \Rightarrow \boxed{\frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2}$$

$$C_{P,1} = C_{P,2} \Rightarrow \frac{P_1}{\rho_1 n_1^3 D_1^5} = \frac{P_2}{\rho_2 n_2^3 D_2^5} \Rightarrow \boxed{\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{n_2}{n_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5}$$



Similar pumps should also have same efficiency, $\eta_1 = \eta_2$, because $\eta \approx \eta(C_Q)$. In practice, larger pumps are more efficient because they have larger Reynolds number, smaller roughness parameter and clearance ratio.

Worked example 4

A company is using a 5.9" centrifugal pump from MP Pumps (performance below) to mobilise $Q = 10\ m^3 \cdot h^{-1}$ of tomato paste, density $\rho = 850\ kg \cdot m^{-3}$. The company needs to halve the head delivered by the pump. The engineer considers buying another pump from the same series and operate at the same speed. Using pump similarity laws, calculate the size of the new pump that should be bought.

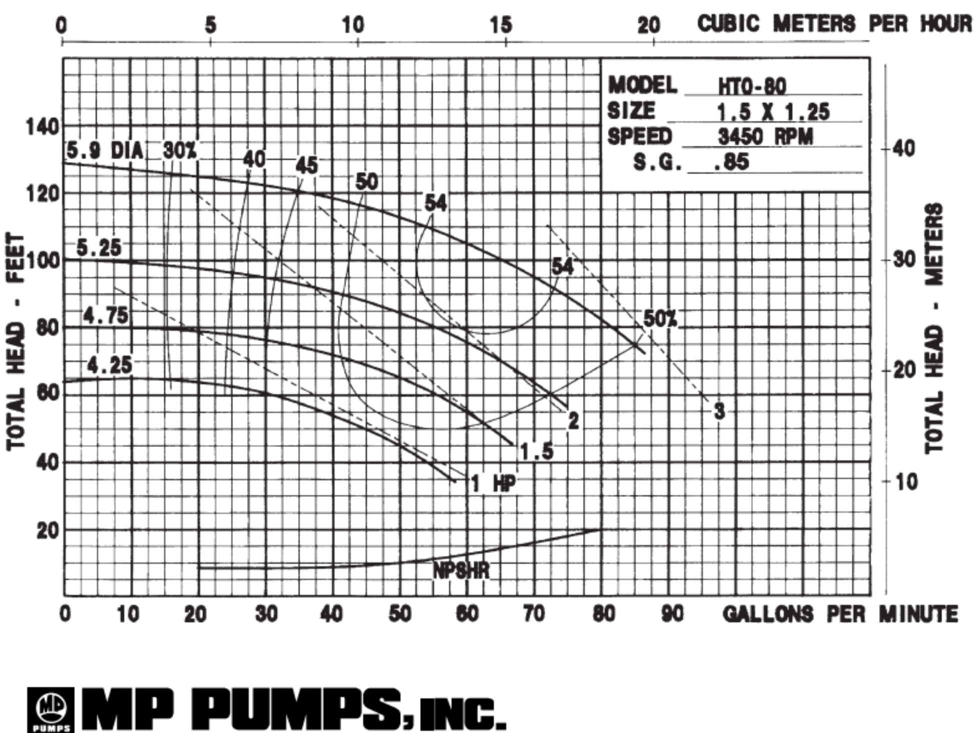
Solution

$$\frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

$$n_2 = n_1$$

$$\Rightarrow \frac{H_2}{H_1} = \left(\frac{D_2}{D_1}\right)^2$$

$$\Rightarrow D_2 = D_1\sqrt{\frac{H_2}{H_1}} = 5.9 * \sqrt{\frac{1}{2}} = 4.7''$$



Mixed- and axial-flow pumps

Centrifugal pumps are high-head, low-discharge machines, therefore not suitable when high flow rates (and low head) are required.

Example: what would be the pump size and speed at BEP for a pump of the same family as those of Worked example 3, that has to deliver 360 m^3/min of water at 15 °C with a head of 7.5 m?

At BEP:

$$C_{H^*} = \frac{gH}{n^2D^2} = 4.7 \Longrightarrow H^* = \frac{4.7n^2D^2}{g}$$

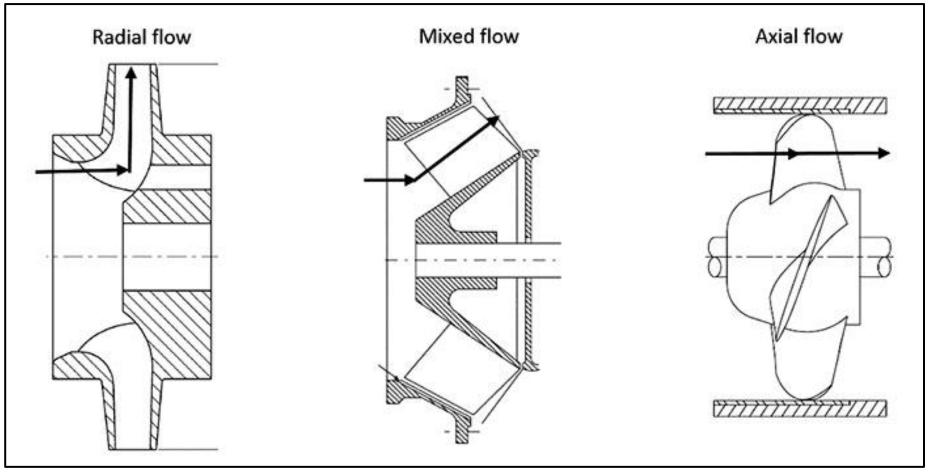
$$C_{Q^*} = \frac{Q^*}{nD^3} = 0.118 \Longrightarrow Q^* = 0.118nD^3$$

Solution of the two equations above yields: $n = 62\ rpm$ and $D = 3.69\ m$.

Look at the solution: you need a huge impeller, almost 4 m of diameter, which will rotate extremely slowly, about one revolution per second. This happens because centrifugal pumps are not designed for large flow rates and low head.

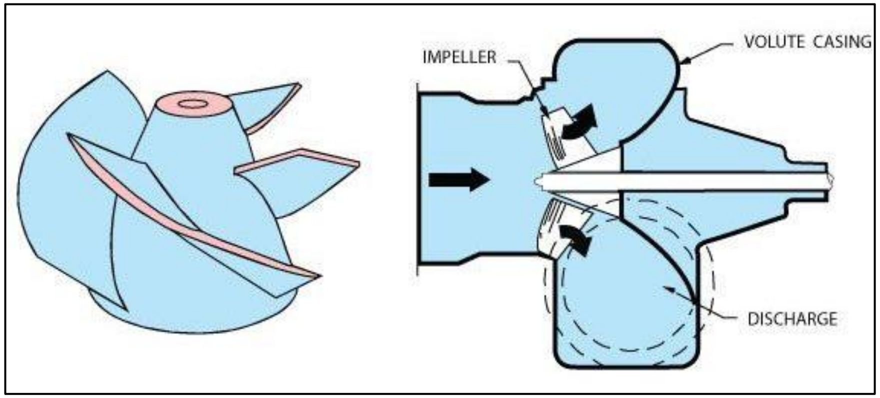
Mixed- and axial-flow pumps

For high flow rates (and low head), mixed-flow and axial-flow pumps are preferred: the flow passes through the impeller with an axial-flow component and less centrifugal component.



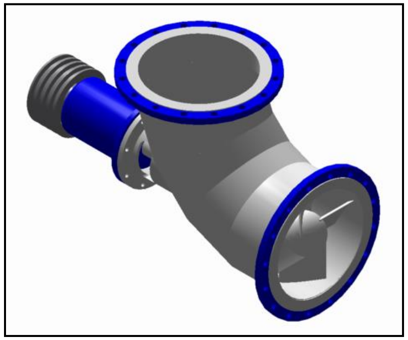
https://www.pumpsandsystems.com/what-difference-between-centrifugal-rotodynamic-pumps

Mixed-flow pump: the blades are shaped to transmit both radial and axial momentum to the fluid

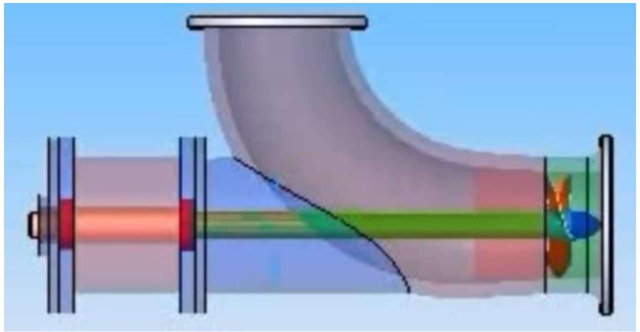


https://savree.com/en/product/centrifugal-pump/

Axial-flow pump: the blades only provide axial momentum



https://www.youtube.com/watch?v=p4pgIS5Ch-M&feature=emb_logo (video)



The specific speed

Given the specific application (required head and flow rate), how do we decide whether to use a centrifugal, mixed-, or axial-flow pump?

This is done based on the **specific speed**, a nondimensional shaft speed obtained by eliminating the diameter between C_Q and C_H, evaluated at BEP conditions:

$$N_s' = \frac{C_{Q^*}^{1/2}}{C_{H^*}^{3/4}} = \frac{n(Q^*)^{1/2}}{(gH^*)^{3/4}}$$

where $n\ [rad/s], Q\ [m^3/s], H\ [m]$. There exists also a 'lazy' dimensional version:

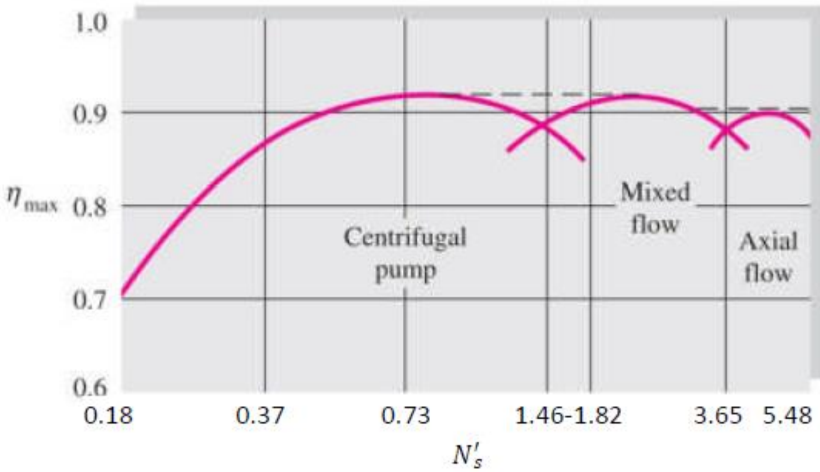
$$N_s = \frac{(n\ [rpm])\ (Q^*\ [US\ gall/min])^{1/2}}{(H^*\ [ft])^{3/4}}$$

where g does not appear at the denominator. Using the conversion factors $1\ rad/s = 9.55\ rpm, 1\ m^3/s = 15850\ US\ gal/min, 1\ m = 3.28\ ft$, and $g = 9.81\ m/s^2$, you obtain the conversion factor:

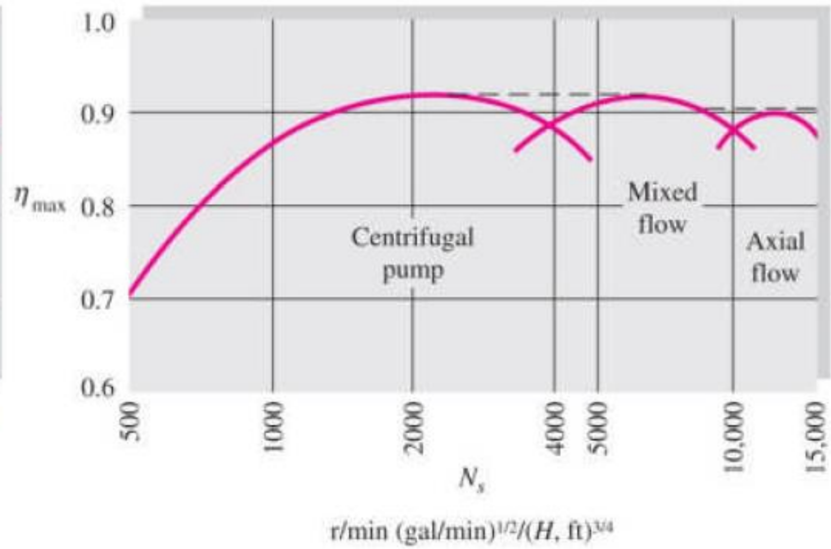
$$N_s' = N_s \times \frac{1}{9.55} \times \left(\frac{1}{15850}\right)^{1/2} \times \left(\frac{1}{1/3.28}\right)^{3/4} \times \left(\frac{1}{9.81}\right)^{3/4} = \frac{N_s}{2734}$$

The specific speed

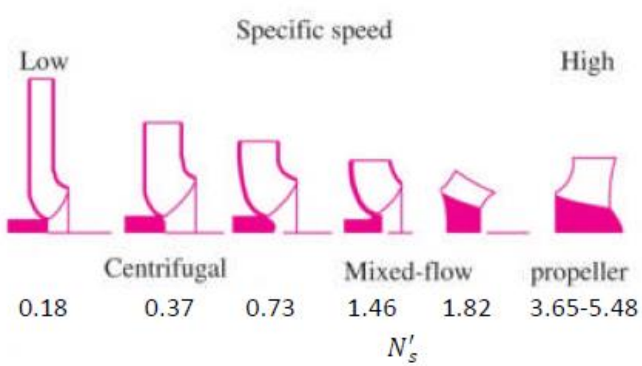
Based on the values of N_s' (or N_s), we can choose the dynamic pump that yields the best efficiency by using the charts below.

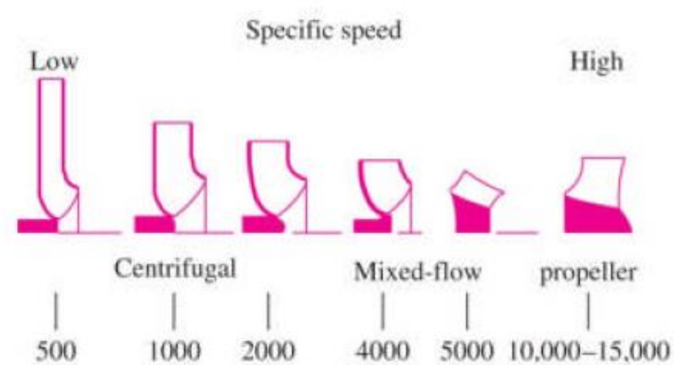


(a)



(a)



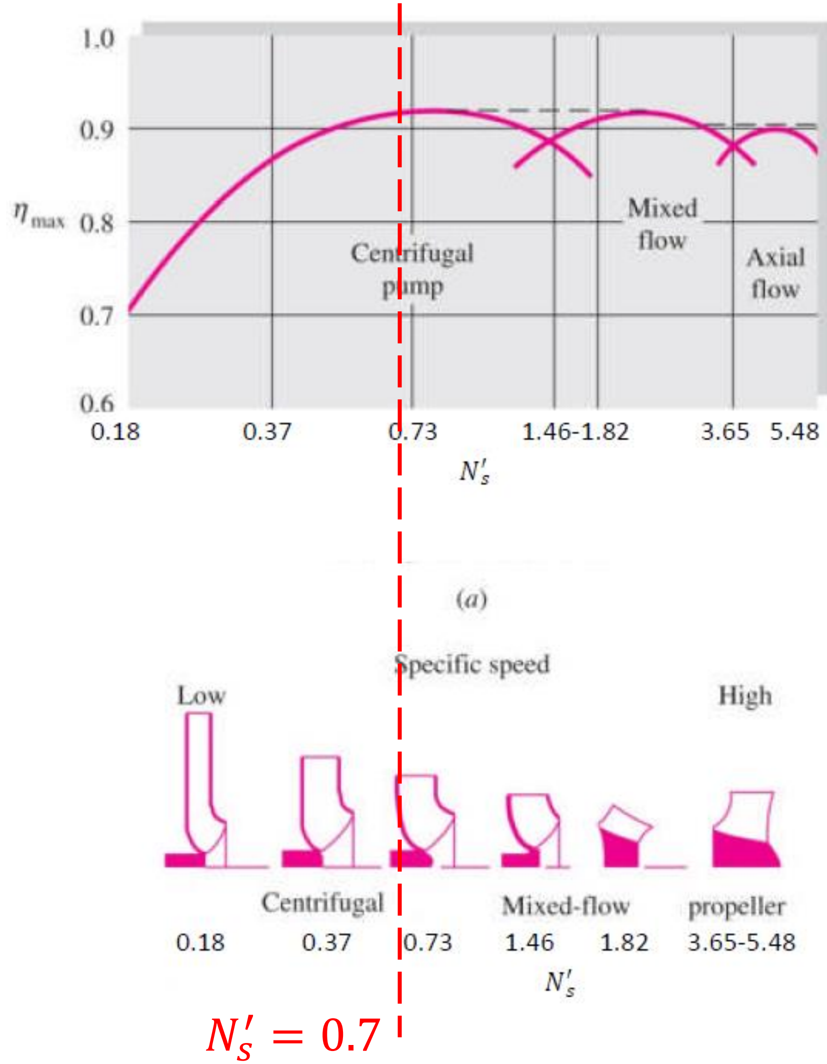


Worked example 5

If a pump has the following values at BEP: C_{H*}= 0.163 and C_{Q*}= 0.0325, what is the non-dimensional specific speed? What rotary pump type does this correspond to?

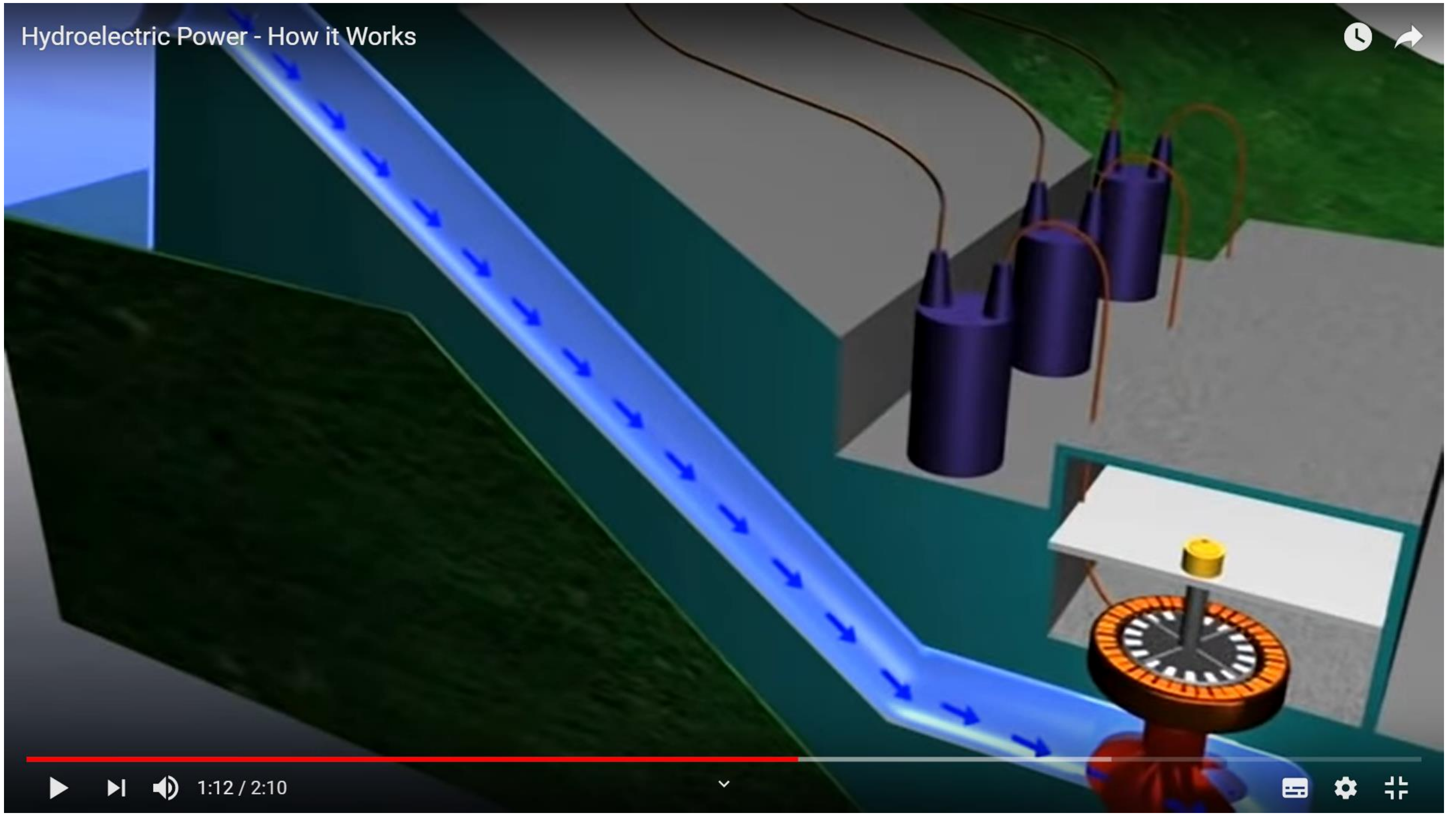
$$N_s' = \frac{C_{Q^*}^{1/2}}{C_{H^*}^{3/4}} = \frac{0.0325^{1/2}}{0.163^{3/4}} = 0.7$$

Centrifugal pump



Turbines

Turbines extract energy from the fluid, taking it from a higher pressure (head) state to a lower pressure (head) state, e.g. to drive an electrical generator.



https://www.youtube.com/watch?v=OC8Lbyeyh-E

Reaction and impulse turbines

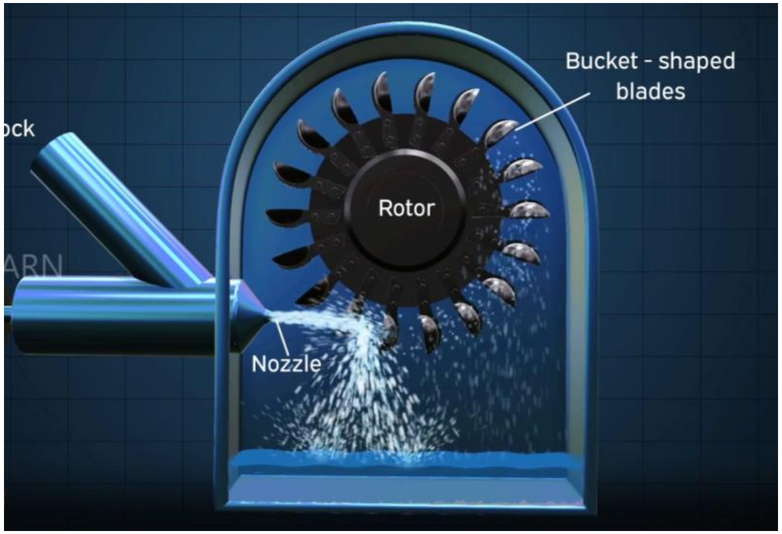
There exist two types of turbine:

- **Reaction turbine**: the fluid fills the blade passages and the pressure change occurs within the impeller. We will see radial (Francis), mixed-flow (Francis) and axial turbines.
- **Impulse turbine**: the high pressure of the flow is converted into a high-speed jet that strikes the blades at one position as they pass by. We will only see the Pelton turbine.

https://www.youtube.com/watch?v=Ack9SPzL8zw

Reaction turbine

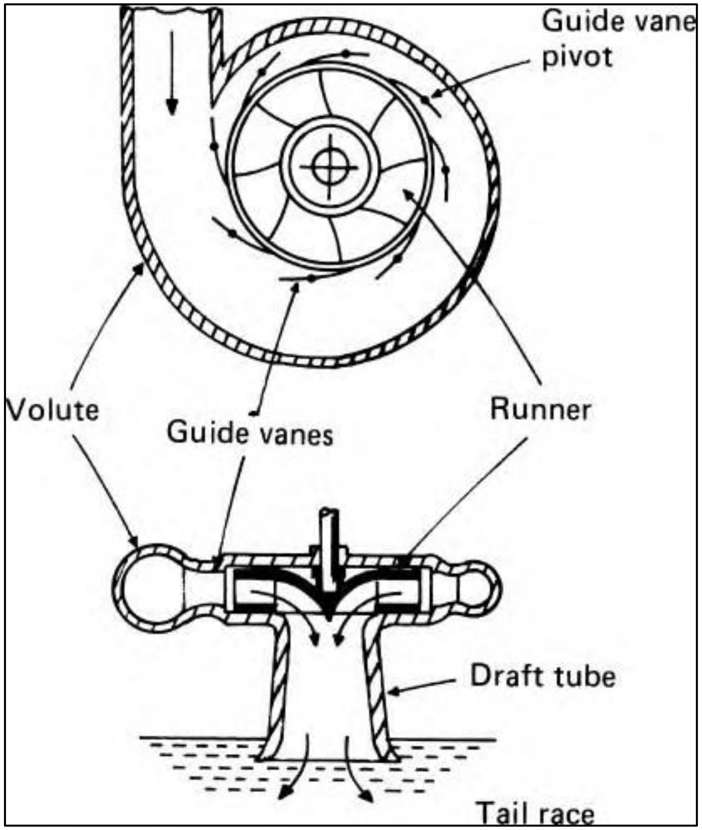
https://www.youtube.com/watch?v=XzopmHnXqwM



Impulse turbine

Reaction turbine

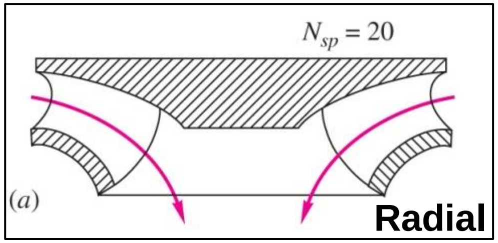
Reaction turbines are, in principle, similar to dynamic pumps, but the fluid travels in the opposite direction. The fluid enters the *volute* casing, its direction is adjusted by stationary *guide vanes* and then transfers its energy to the *runner* (the rotor).

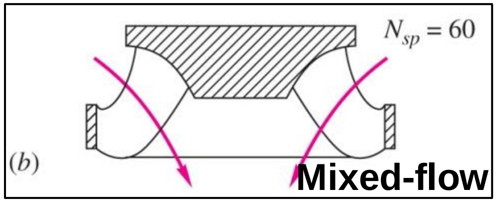


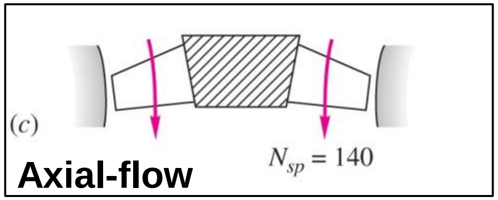
Schematic of a Francis (radial) turbine, from B. Massey

From the centre of the runner, the fluid leaves along the axial direction via the *draft tube*.

- Reaction turbines are optimal for low-head and high-flow conditions (same as dynamic pumps).
- Radial, mixed-flow and axial-flow turbines exist. The difference is in the runner geometry:



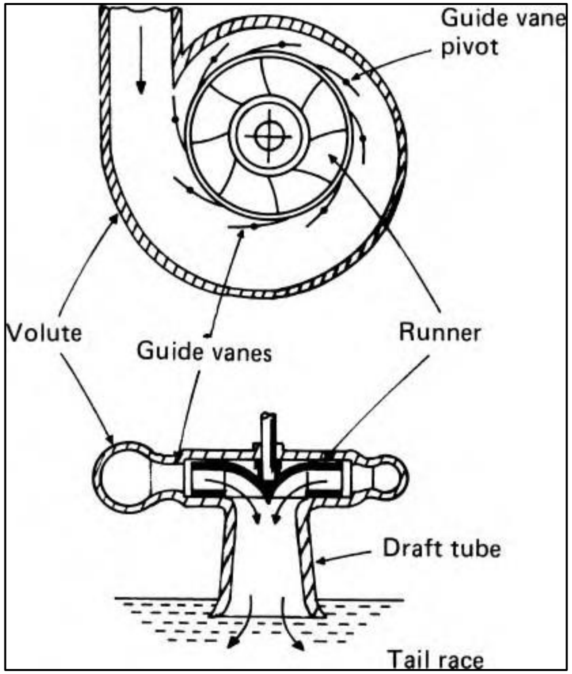




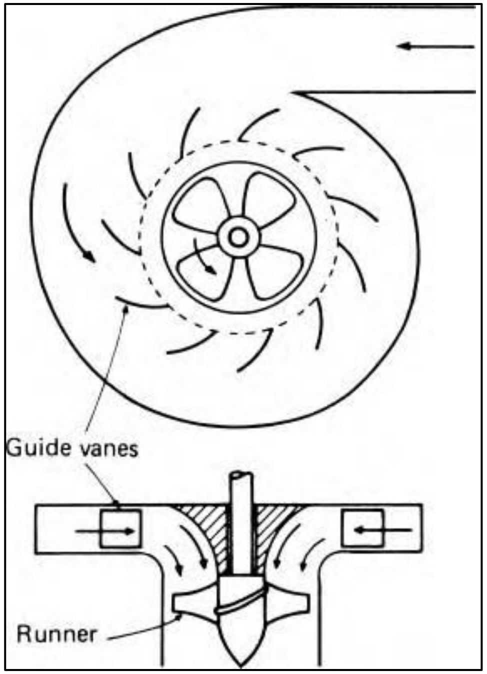
Reaction turbine

- Radial turbines are preferred at low flow rates (and high head), whereas mixed- and axial-flow turbines are preferable at high flow rates (low head).
- Radial and mixed-flow turbines are called *Francis* turbines; axial-flow turbines are called *propeller* turbines or *Kaplan* turbines when the blades of the propeller are adjustable.

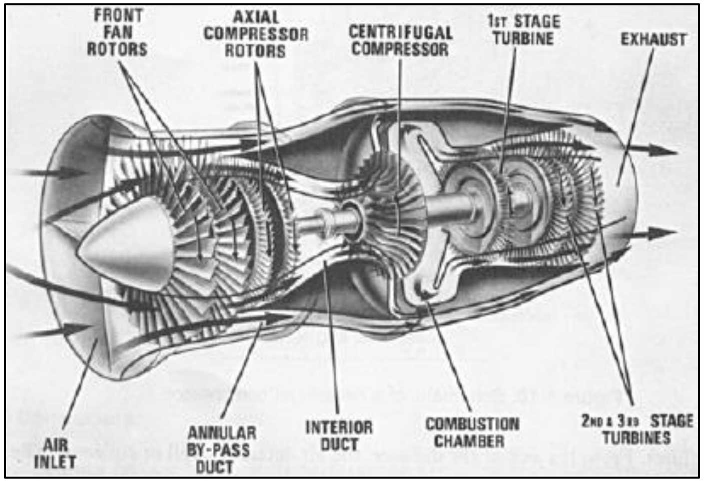
Francis radial turbine



Propeller turbine



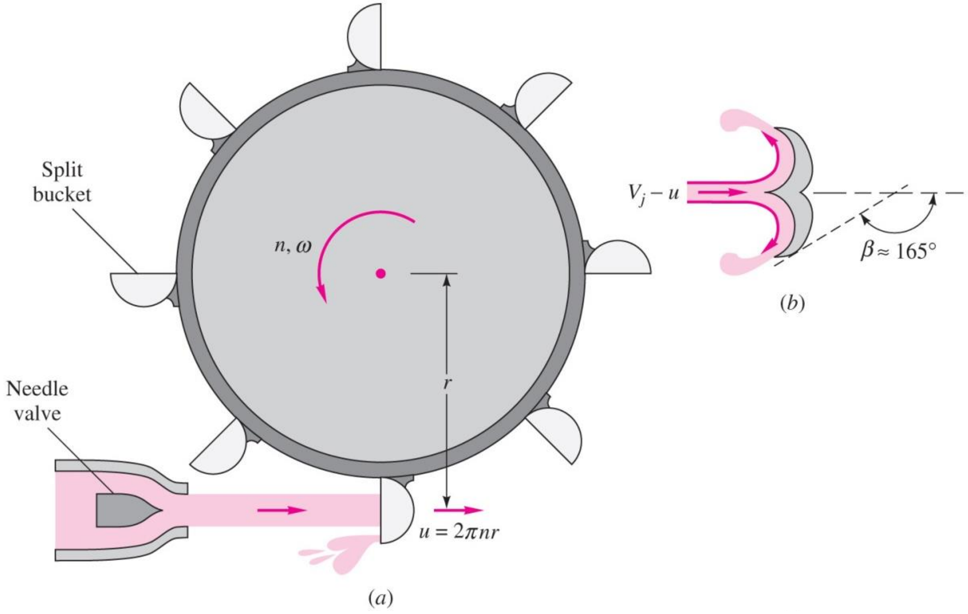
Jet engines employ axial gas turbines

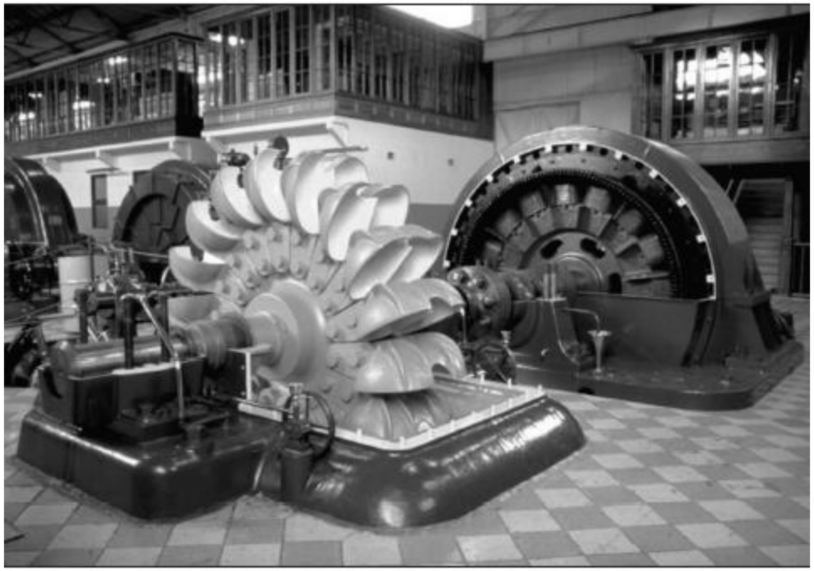


Impulse turbine

In an impulse turbine, one (or more) small nozzle converts the high-pressure fluid into a high-speed jet at atmospheric pressure, injected tangential to the rotor. The rotor is equipped with spoon-shaped *split buckets*. The jet strikes the buckets and therefore transfer momentum to the rotor.

- Impulse turbines are also known as *Pelton* turbines.
- They are used for high-head and low-flow conditions.





Dimensionless turbine performance

The dimensional analysis applied to turbines follows exactly that developed for pumps, and the same nondimensional groups are formed:

$$C_H = \frac{gH}{n^2 D^2}$$ Head coefficient

$$C_P = \frac{P}{\rho n^3 D^5}$$ Power coefficient

$$C_Q = \frac{Q}{n D^3}$$ Capacity coefficient

However, while in a pump we introduce power to create a head so that $\eta = \rho Q g H / P$, in a turbine we introduce fluid head to create power, and therefore the efficiency writes as:

$$\eta = \frac{P}{P_w} = \frac{P}{\rho Q g H}$$

Furthermore, while in pumps performance are given as a function of the flow rate, $C_H \approx f(C_Q)$ and $C_P \approx f(C_Q)$, for turbines performance are given as a function of the output power P, $C_H \approx f(C_P)$ and $C_Q \approx f(C_P)$, so that also $\eta \approx f(C_P)$.

Dimensionless turbine performance

$$C_H = \frac{gH}{n^2 D^2} \quad \text{Head coefficient}$$

$$C_P = \frac{P}{\rho n^3 D^5} \quad \text{Power coefficient}$$

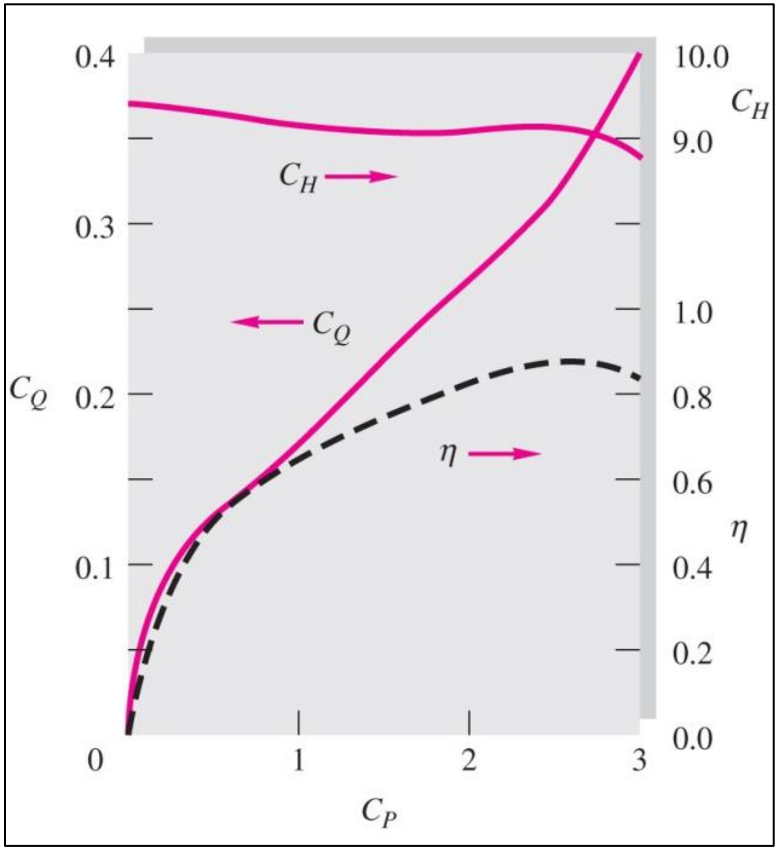
$$C_Q = \frac{Q}{n D^3} \quad \text{Capacity coefficient}$$

$$\eta = \frac{P}{P_w} = \frac{P}{\rho Q g H}$$

For turbines performance are given as a function of the output power P, $C_H \approx f(C_P)$ and $C_Q \approx f(C_P)$, so that also $\eta \approx f(C_P)$.

The maximum efficiency point for turbines is called *normal power*.

Performance curves for a Francis radial turbine



Power specific speed for turbines

Equivalent to the specific speed for pumps, the **power specific speed** for turbines is a parameter that allows us to choose the turbine type based on the available head and output power. The power specific speed is a nondimensional shaft speed, obtained by eliminating the diameter between C_P and C_H, evaluated at normal power (best efficiency) conditions:

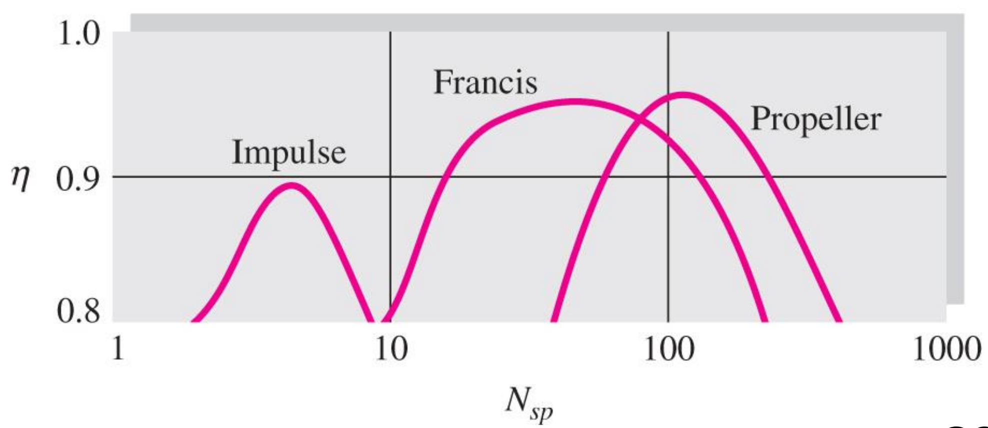
$$N'_{sp} = \frac{C_{P^*}^{1/2}}{C_{H^*}^{5/4}} = \frac{n(P^*)^{1/2}}{\rho^{1/2}(gH^*)^{5/4}}$$

where, usually, $n\ [rad/s], H\ [m], P\ [W]$. There exists also a dimensional version:

$$N_{sp} = \frac{(n\ [rpm])\ (P^*\ [hp])^{1/2}}{(H^*\ [ft])^{5/4}}$$

with a conversion factor of:

$$N'_{sp} = \frac{N_{sp}}{43.5}$$



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Wind turbines

Wind turbines extract energy from the wind to drive a shaft that is connected to an electric generator (modern use). Windmills are an example of early wind turbines.

Horizontal axis wind turbine (HAWT)

Vertical axis wind turbine (VAWT)

Horizontal axis wind turbines (HAWT)

The shaft of the turbine is horizontal.

https://www.youtube.com/watch?time_continue=1&v=LNXTm7aHvWc&feature=emb_logo

Vertical axis wind turbines (VAWT)

The shaft of the turbine is vertical.

https://www.youtube.com/watch?time_continue=1&v=4uJCiJmVbjM&feature=emb_logo

HAWT vs VAWT

HAWT

- More efficient than VAWT
- Taller than VAFT, air speed is higher and therefore gives more power

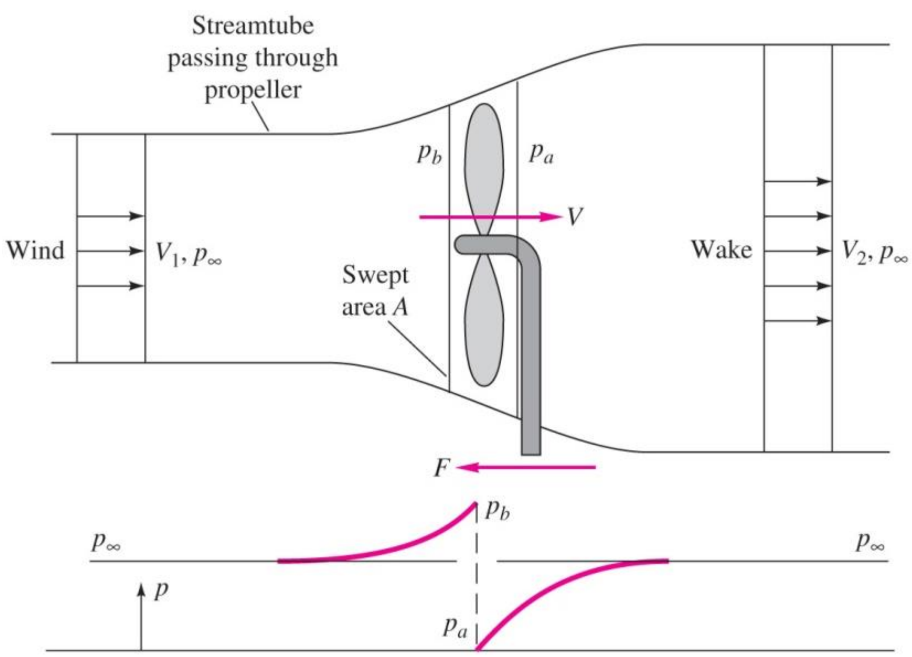
VAWT

- Smaller than HAFT, and therefore cheaper
- Because of the vertical shaft, gearbox and generator are at ground level
- Easier to construct and change parts
- More silent than HAWT
- Can be used in places where the wind changes frequently

Idealised wind turbine theory (for HAWT)

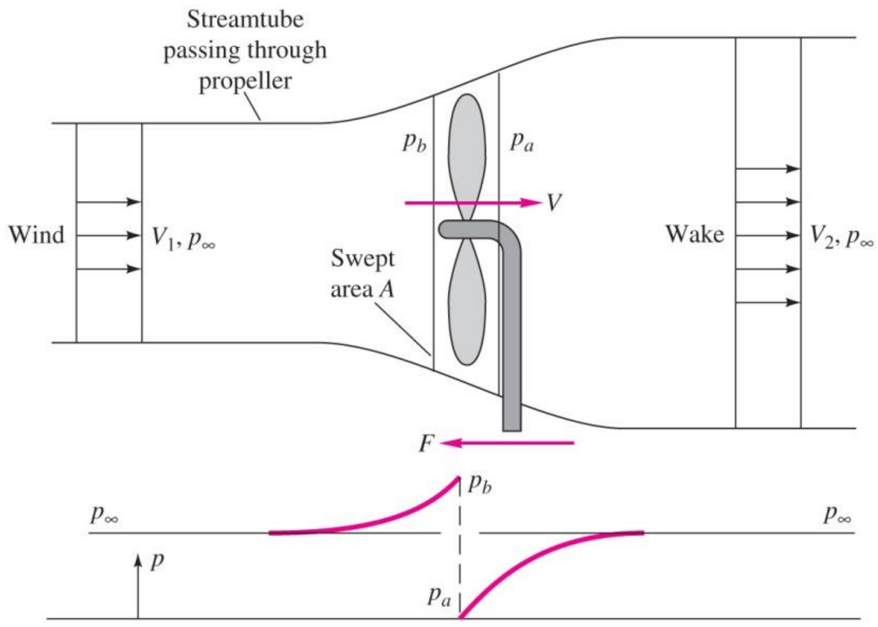
Theory first developed by Betz (1920) to find the maximum power that a wind turbine can extract from the wind.

- It models the wind flow as a streamtube across the propeller.
- The propeller is modelled as an actuator disk, which creates a pressure discontinuity $p_b - p_a$ in the fluid.
- The pressure rises to p_b just before the disk and drops to p_a just after, returning to the free-stream pressure in the far wake.
- The wind exerts a force on the propeller, which is balanced by an opposite force F on its support.



Idealised wind turbine theory (for HAWT)

We assume frictionless flow and apply the linear momentum equation (TF1, p. 99) on a control volume between sections 1 and 2:



$$-F = \dot{m}(V_2 - V_1)$$

$\dot{m} = \rho Q = \rho V A$: mass flow rate

$-F$: force exerted on the fluid

Momentum conservation between a and b:

$$-F + p_b A_b - p_a A_a = \dot{m}(V_a - V_b)$$

But $A_a \approx A_b = A$ and, by continuity, $\rho V_a A_a = \rho V_b A_b \Longrightarrow V_a \approx V_b = V$, so that:

$$-F + (p_b - p_a)A = \dot{m}(V_a - V_b) = 0 \Longrightarrow F = (p_b - p_a)A = \dot{m}(V_1 - V_2)$$

This tells us that the propeller introduces a sudden pressure drop in the fluid, $(p_b - p_a)$, that gives rise to a net force F acting on the propeller. As a consequence of this force, the fluid slows down between the inlet and the far wake, $V_1 > V_2$.

Idealised wind turbine theory (for HAWT)

$$F = (p_b - p_a)A = \dot{m}(V_1 - V_2)$$

We now apply Bernoulli between 1 and b:

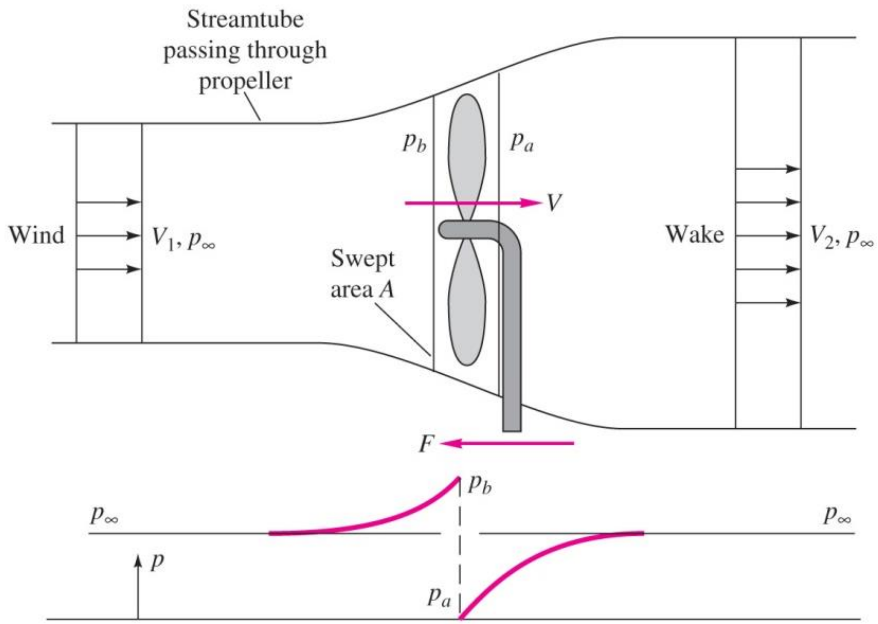
$$p_\infty + \frac{1}{2}\rho V_1^2 = p_b + \frac{1}{2}\rho V^2$$

And between a and 2:

$$p_a + \frac{1}{2}\rho V^2 = p_\infty + \frac{1}{2}\rho V_2^2$$

Subtracting these:

$$\frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2 = p_b - p_a$$



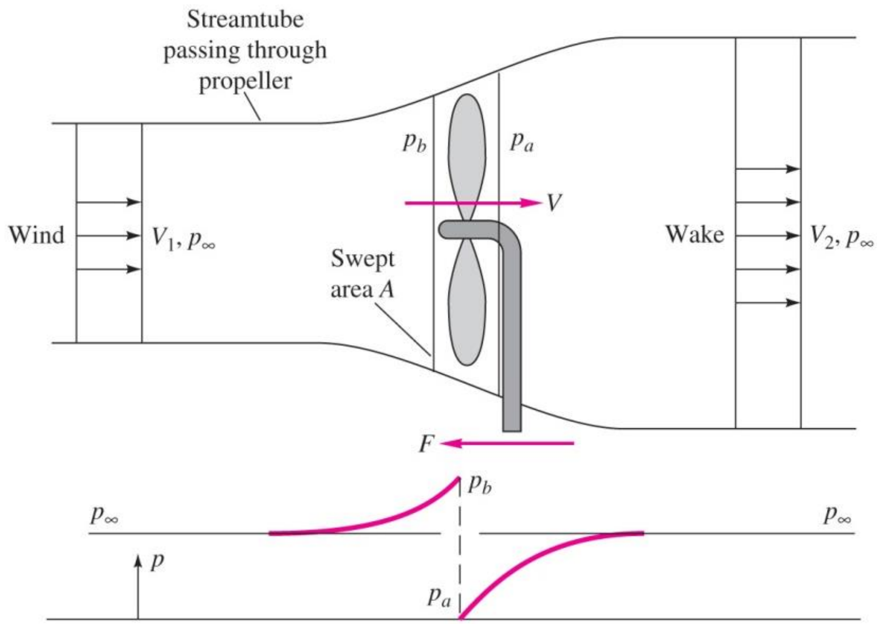
From the previous slide: $(p_b - p_a)A = \dot{m}(V_1 - V_2) \Rightarrow p_b - p_a = \frac{\dot{m}}{A}(V_1 - V_2) = \rho V(V_1 - V_2)$

And therefore:

$$p_b - p_a = \rho V(V_1 - V_2) = \frac{1}{2}\rho(V_1 + V_2)(V_1 - V_2)$$

➡ $V = \frac{V_1 + V_2}{2}$ The velocity through the disk equals the average of wind and far-wake speeds

Idealised wind turbine theory (for HAWT)



We have obtained:

$$F = (p_b - p_a)A = \dot{m}(V_1 - V_2)$$

$$V = \frac{V_1 + V_2}{2}$$

We can now express the power extracted

by the propeller:

$$P = FV = \dot{m}(V_1 - V_2)V = \rho A V^2 (V_1 - V_2)$$
$$= \rho A \left(\frac{V_1 + V_2}{2}\right)^2 (V_1 - V_2) = \frac{1}{4}\rho A (V_1^2 - V_2^2)(V_1 + V_2)$$

For a given speed of the wind V_1, what is the maximum possible power? This depends on V_2, because $P = P(V_2)$. We can find the max P by requiring that $dP/dV_2 = 0$:

$$\frac{dP}{dV_2} = 0 \Longrightarrow \frac{d(V_1^3 + V_1^2 V_2 - V_2^2 V_1 - V_2^3)}{dV_2} = 0 \Longrightarrow V_1^2 - 2V_2V_1 - 3V_2^2 = 0$$

Solve for V_2: solutions are $V_2 = V_1/3$ and $V_2 = -V_1$, the latter not plausible.

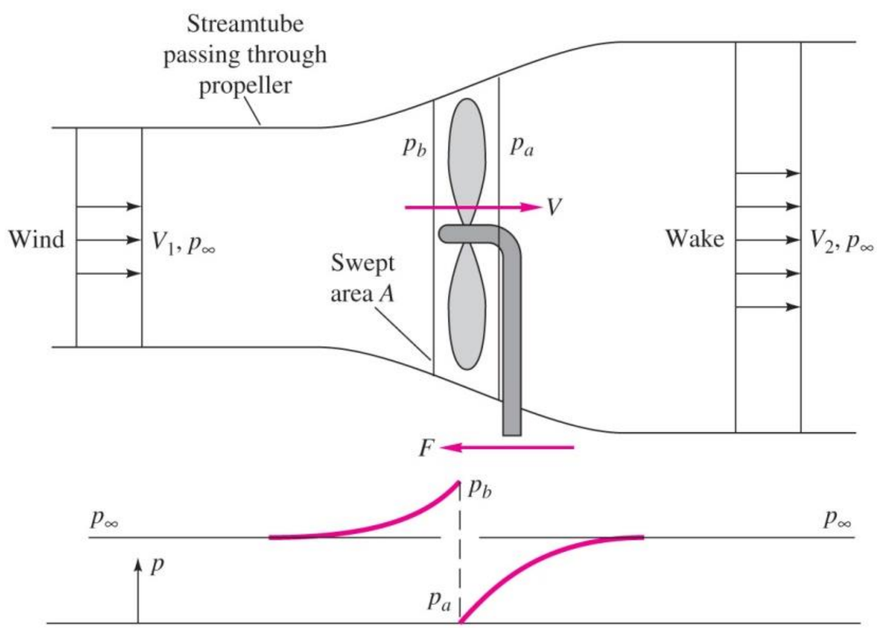
Idealised wind turbine theory (for HAWT)

$$\frac{dP}{dV_2} = 0 \Rightarrow V_1^2 - 2V_2V_1 - 3V_2^2 = 0$$

Solve for V_2: solutions are $V_2 = V_1/3$ and $V_2 = -V_1$, the latter not plausible.

Therefore, $P = P_{max}$ when $V_2 = V_1/3$:

$$P_{max} = \frac{1}{4}\rho A(V_1^2 - V_2^2)(V_1 + V_2)$$
$$= \frac{1}{4}\rho A\left(V_1^2 - \frac{V_1^2}{9}\right)\left(V_1 + \frac{V_1}{3}\right) = \frac{8}{27}\rho A V_1^3$$



$$P_{max} = \frac{8}{27}\rho A V_1^3$$

This is the **maximum possible power** that the propeller can extract from the wind

The **maximum available power** from the wind approaching the propeller is given by the product of the specific kinetic energy and mass flow rate:

$$P_{avail} = \frac{1}{2}\dot{m}V_1^2 = \frac{1}{2}\rho A V_1^3$$

Idealised wind turbine theory (for HAWT)

$$P_{max} = \frac{8}{27}\rho A V_1^3$$

$$P_{avail} = \frac{1}{2}\dot{m}V_1^2 = \frac{1}{2}\rho A V_1^3$$

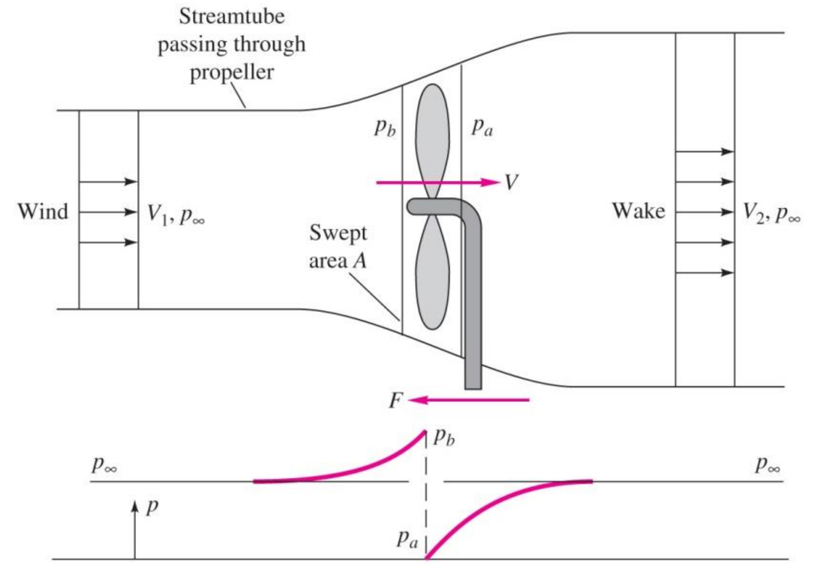
We define as **power coefficient** the ratio of power extracted P and max available:

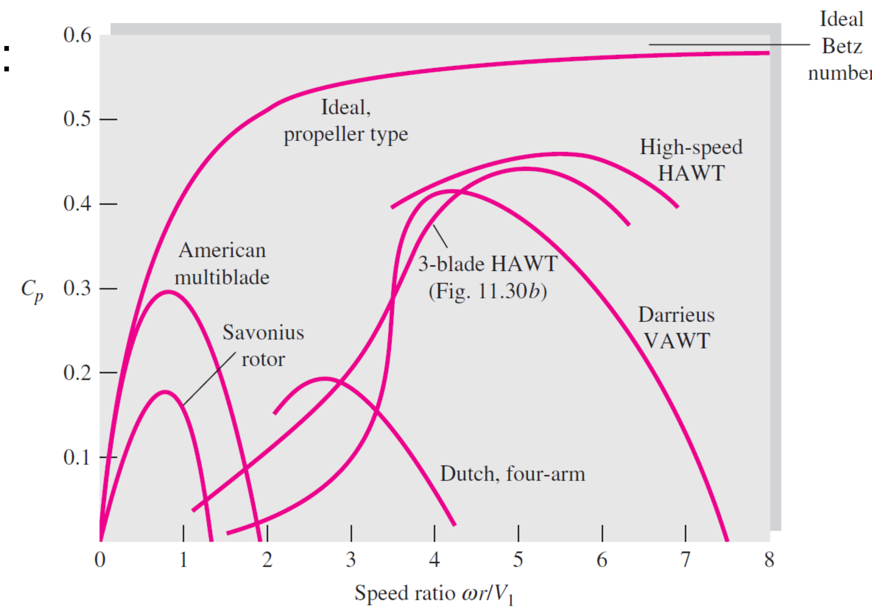
$$C_P = \frac{P}{P_{avail}}$$

This leads us to the maximum possible efficiency for an ideal, frictionless wind turbine:

$$C_{P,max} = \frac{P_{max}}{P_{avail}} = \frac{\frac{8}{27}\rho A V_1^3}{\frac{1}{2}\rho A V_1^3} = \frac{16}{27} = 0.593$$

0.593 is the **Betz number** and serves as an ideal with which to compare the actual performance of real wind turbines.





Topic 5 – Turbomachinery

Now you should:

- Know the difference between devices that *add* energy to a fluid and those that *remove* energy
- Know the two difference between the two main *pump classifications* (positive displacement and dynamic)
- *Positive displacement pumps.*
  - Be able to name, sketch and describe the operation of the main types of PD pumps
  - Be able to give the main advantages and disadvantages of PD pumps
- *Dynamic pumps.*
  - Be able to name, identify and explain the function of the main parts of a centrifugal pump (eg impeller, eye, outlet pipe etc)
  - Be able to sketch and name the three blade types for a centrifugal pump and know their advantages/disadvantages
  - Recognise and be able to use the expression for power delivered to the pump fluid (water power, $P_w = \rho Q g H$)
  - Recognise and be able to use the expression for pump input power ($P = \omega T$)
  - Recognise and be able to use the expression for pump efficiency

Topic 5 – Turbomachinery

Learning outcomes:

- *Dynamic pumps (continued).*
  - Be able to write down and explain the main causes of inefficiency in a centrifugal pump
  - Be able to sketch and describe all the main features of a pump performance curve
  - Be able to read data from a manufacturer’s pump performance chart (flowrate, head, efficiency, NPSH)
  - Be able to describe the cause and main effects of cavitation in a pump
  - Be able to perform NPSH calculations using EBE
  - Recognise and be able to use the 5 pump non-dimensional groups
  - Recognise and be able to use pump similarity rules

- *Mixed and axial flow pumps*
  - Know when a mixed or axial flow pump might be the best choice
  - Be able to calculate specific speed
  - Be able to use the specific speed chart to work out which pump type is best for a given application

Learning outcomes:

- *Turbines*
  - Be able to articulate the main difference between reaction and impulse turbines
  - For reaction turbines be able to explain where radial, mixed and axial flow configurations are best
  - Recognise and be able to use the turbine non-dimensional groups
  - Be able to calculate power specific speed
  - Be able to use the power specific speed chart to determine the most appropriate turbine type

- *Wind turbines*
  - Be able to sketch and describe HAWT and VAWT
  - Be able to articulate the main advantages and disadvantages of HAWT and VAWT
  - Be able to explain using the Betz actuator disk approach with equations, diagrams and text why there is a maximum power coefficient for a wind turbine
  - Know $C_{P,max}$ for an ideal, frictionless wind turbine

Seminar

Worked example 6

Exam 2016/17: fluids long question

A centrifugal pump with impeller diameter of 0.15m rotates at 40 Hz and delivers 5 litres per second of water with a total head of 41m. The pump is operating at best efficiency point at 85%. The inlet and outlet pipe diameters are both 50mm.

(a) Calculate the power output and the power of the motor driving the pump.

(b) At 48Hz, with the flow rate at 5.5 litres per second and inlet pressure of 0.8 bar, cavitation occurs. Calculate the velocity at inlet and the NPSH for this pump.

(c) Calculate the specific speed, and describe briefly why this parameter is useful in selecting a pump.

(d) Briefly describe the purpose of dimensionless pump characteristic curves and why a particular pump application may vary slightly from the expected characteristics.

For (b), consider that the vapor pressure of the water at the inlet is 2337 Pa.

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Worked example 6

Solution

Worked example 6

Solution

Worked example 6

Solution

Worked example 7

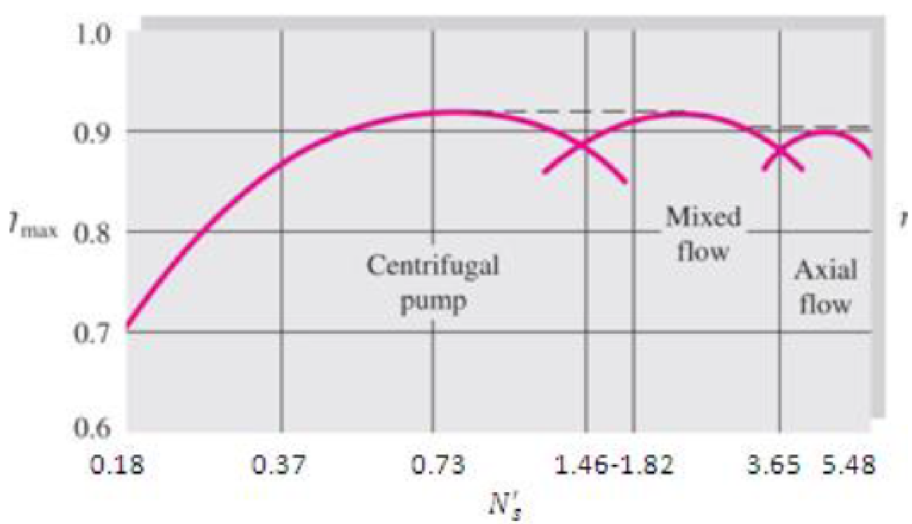
Exam 2018/19: fluids long question

The torque T delivered by a water pump depends on the volumetric flowrate Q, pressure P, head H, and angular speed ω.

(a) By using the Π theory, determine how many dimensions and dimensional variables are involved in this situation.

(b) Calculate one of the dimensionless ratios by taking T as a base variable and H, ω and P as repeating variables.

(c) This pump reaches its maximum efficiency when its discharge is $0.11 m^3/min$, shaft speed 2500 r.p.m. (revolutions per minute) and pressure head is 20m of water. Use Figure Q14 to judge what kind of pump this is.



Worked example 7

Solution

Worked example 7

Solution

Worked example 8

Exam 2016/17: fluids short question

A pump has impeller diameter of 15 cm and shaft speed of 3000 revolutions per minute, with a delivery 0.12 m^3/min of water. Calculate the diameter of the impeller of a similar pump to deliver twice the flow rate at the same speed.

Solution

Worked example 9

Exam 2017/18: fluids long question

Steam is injected through a nozzle in a large scale turbine with a velocity U equal to 100 m/s. The steam conditions are: T = 400°C, pressure is 0.1 MPa, and specific heat capacity of the steam, c_p, is 1930 J/kg K and specific heat at constant volume, c_v, is 1469 J/kgK. The nozzle has an area, A, equal to 0.5 m^2.

(a) ~~Calculate the speed of sound in steam and determine the Mach number~~.

(b) The turbine is rotating at 5 rad/s and the turbine diameter, D, is equal to 10 m. Calculate the capacity coefficient.

(c) It is necessary to replace the turbine with a new similar turbine having a diameter, D, equal to 5m. The new turbine uses the same nozzle and the same flow rate as the old turbine. Calculate its rotational speed.

Worked example 9

Solution

(b)

(c)

Worked example 10

Exam 2015/16: fluids long question

Turbomachinery and similarity analysis:

A pump with an impeller diameter $D = 0.2m$ that operates at $n = 25\,rad/s$ (i.e. $n = \omega$), delivers $Q = 0.03m^3/s$ of water and a total head $H = 67m$ under a Power efficiency of $\eta = 72\%$. The inlet pressure of the pump is $p_i = 117.6 \times 10^3 \text{kg/ms}^2$ (i.e. N/m^2)

The inlet and outlet pipes of the pump are identical with the same radius, the inlet and outlet pipe area is $A = A_i = A_o = \pi r^2$. Also the inlet and outlet pipes are located at the same vertical position, i.e. $z_i = z_o$.

For water at $T = 20\,°\text{C}$, $\rho = 10^3\,kg/m^3$
The gravitational acceleration is $g = 9.8\,m/s^2$.

(a) Find the Power delivered by the pump to the liquid (*water power*) and the Power driving the pump (*brake power*).

[2 marks]

(b) Knowing that the inlet and outlet pipes are identical and located at the same vertical position, find the pressure at the outlet of the pump given that the total head delivered by the pump is $H = (H_{To} - H_{Ti}) = 67m$, with H_T as the hydraulic head constant in Bernoulli equation.

[2 marks]

Worked example 10

Exam 2015/16: fluids long question

(c) Find the operating speed n_2 (angular velocity), the total head delivered H_2, the Power delivered by the pump to the liquid, P_{w2}, the efficiency η_2, and the Power driving the pump, P_2, of a dynamically similar pump to the one used in parts a) and b) but with an impeller of diameter $D_2 = 0.18m$ instead of $D_1 = 0.2m$, which will be used in series, i.e. one pump behind the other $(Q_2 = Q_1 = 0.03m^3/s)$, in order to further increase the pressure at the outlet of the pump system

[4 marks]

Worked example 10

Solution

(a)

(b)

(c)

Worked example 10